Using a De-Convolution Window for Operating Modal Analysis

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ABSTRACT

Operating Modal Analysis (OMA) has been applied in a number of cases recently for finding the experimental modal parameters from structures when their excitation forces cannot be measured. Without force measurement, a classical Experimental Modal Analysis (EMA), which relies on the application of modal parameter estimation, or curve fitting methods to a set of Frequency Response Function (FRF) measurements, cannot be performed.

In this paper, we show that the application of a de-convolution window to cross power spectrum data yields the effective recovery of the FRFs involving each vibration response signal. A set of recovered FRFs can then be curve fit using classical FRF-based curve fitting methods to identify the experimental modal parameters of the structure. Use of the de-convolution window is illustrated in an example that uses a FRF matrix model to calculate structural responses to simulate a multi-reference OMA.

INTRODUCTION

Experimental Modal Analysis (EMA), underwent a revolutionary change during the early 1970's with the implementation of the Fast Fourier Transform (FFT) in computer-based FFT analyzers [1]. Modal parameter estimation is a key step in FFT-based EMA. This one step, also called curve fitting, has probably received more attention than any other during the past 30 years. Numerous methods have been developed, and the technical literature contains 100's of papers documenting many different approaches.

Two completely different approaches to modal testing; one that relies on carefully controlled shaker excitation [3], and the other that strongly suggests that artificial excitation is not required at all [4], have been the topic of much discussion recently. The first approach is commonly called an EMA, and the second an OMA. Regardless of whether artificial excitation is used or not, both approaches rely heavily on modal parameter estimation. Both articles [3] & [4] devote the majority of their discussion to this subject.

Modal analysis is used to characterize resonant vibration in machinery and structures. A mode of vibration is defined by three parameters; ***modal frequency***, ***modal damping***, and a ***mode shape***. Modal parameter estimation, or curve fitting is the process of determining these parameters from experimental data.

Furthermore, it can be shown that a set of modal parameters ***completely characterizes*** the dynamic properties of a structure. This set of parameters is called a ***modal model***.

A modal model will be used later in this paper to perform a ***"round trip"*** exercise. The modal model will be used to calculate the forced random response of a structure, and its acceleration responses will be used to simulate a multi-reference OMA. With application of the de-convolution window and FRF curve fitting to a set of response only Cross spectrum measurements, modal parameters will be recovered from the response only data, and then compared to the parameters of the original modal model.

Which is Better, EMA or OMA?

Although time, budget, and physical constraints will most certainly play a part, the modal testing method that you choose strongly depends on what you intend to do with the modal data. The two most common reasons for performing a modal test are,

1. Trouble shooting a resonance related noise or vibration problem.
2. Verifying and updating a computer generated finite element model.

Trouble shooting a noise or vibration problem only requires enough data to characterize the problem so that a solution can be found. Verifying and updating a finite element model usually requires much more extensive and accurate modal testing.

Finite element analysis (FEA) is commonly used today in the engineering development of most new machines, structures, and products of all kinds. Once a finite element model is validated, it can be used for simulations, calculating stresses and strains, and for investigating the effects of structural modifications on a structure's acoustic or vibration behavior. Since EMA and FEA both yield a set of modes for a structure, modal parameters are used for comparing results, and for updating the FEA model to more closely match the experimental results.

If no artificial excitation is required and excitation forces don't have to be measured, simply acquiring and processing operating, (response only, or output only) data appears to be an easier way to perform a modal test. Simply acquiring the vibration response of a machine while it is operating or being excited in situ is easier than artificially exciting it and simultaneously measuring both the excitation forces and responses.

However, the assumptions required for OMA are more restrictive then when the excitation forces are measured. Therefore, controlling and measuring the excitation forces is the preferred way to do modal testing when possible.

Nevertheless, when the excitation forces cannot be measured, then properly post-processed and curve fitting a set of response only measurements can still provide accurate modal parameter estimates.

What Is Operating Data?

Operating data is what the name implies. It is data that is acquired while a machine or structure is undergoing vibratory motion during its operation or use. For modal parameter estimation, the definition can be extended further.

Operating Data is any vibration data that is acquired without simultaneously acquiring the excitation forces.

Shape Data

Whenever the vibration responses at two or more points and directions (degrees-of-freedom or DOFs) on the surface of a structure are measured, a vibration ***shape*** is defined. That is, a shape defines the magnitude and phase of the motion of one DOF relative to another DOF.

An Operating Deflection Shape (ODS) is the magnitudes and phases of two or more DOFs of operating data acquired from a machine or structure.

Therefore, an ODS defines the relative motion between two or more DOFs on a structure. An ODS can be defined for a specific frequency or for a moment in time [2].

Broad Band Excitation

All excitation forces can be classified as either ***narrow band*** like a single frequency sine wave, or ***broad band***, or a combination of both. The most common broad band signals are random, swept sine or chirp, and transient or impulsive. Variations of these signals include burst random, burst chirp, and random transient [5].

A sine wave is classified as narrow band because its spectrum is very narrow, containing essentially a single non-zero frequency. All broad band signals have a non-zero frequency spectrum over a broad range of frequencies.

Curve Fitting Methods

The two most popular approaches to curve fitting either curve fit a parametric model of the FRF to experimental FRF data, or curve fit a parametric model of an Impulse Response Function (IRF) to experimental IRF data. The Rational Fraction Polynomial (RFP) method is commonly used for curve fitting FRFs, and the Complex Exponential (CE) for curve fitting IRFs.

The FRF and its corresponding IRF form a Fourier transform pair. That is, an IRF is obtained by applying the Inverse FFT to an FRF, and the FRF can be recovered by applying the Forward FFT to the IRF. Therefore, either FRFs or their equivalent IRFs can be curve fit by starting with either one and using the FFT to transform to the other.

Many variations of the RFP and CE methods have been proposed and documented [6], [7]. Both methods of these methods were used for curve fitting the operating data presented in this paper. Other types of curve fitting based on state-space models have also been used for curve fitting operating data [4].

Impulse Response Function

Since the IRF is the Inverse Fourier transform of the FRF, each element of an FRF matrix has and equivalent IRF in the time domain. Modal parameters are therefore estimated from a set of IRFs in the similar way as they are estimated from a set of FRFs.

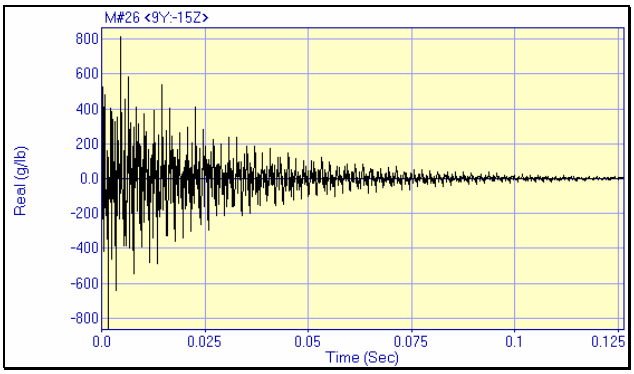

Figure 1. An IRF.

BACKGROUND THEORY

Recall that operating data is acquired in any situation where the excitation forces are not measured. It is possible to curve fit operating data using an FRF or IRF curve fitting model, but ***an assumption regarding the spectrum of the unknown excitation forces is required***.

Roving & Reference Responses

Assume that a certain number of the responses are used a ***Reference*** responses, i.e. they are measured at fixed DOFs through the OMA. The responses that are not fixed during an OMA are referred to as ***Roving*** responses.

A multi-input multi-output (MIMO) FRF matrix model for the ***Roving*** responses can be written as follows,

$$\{X(\omega)\} = [A(\omega)]\{F(\omega)\} \qquad (1)$$

where:

$\{X(\omega)\} =$ **n**-vector of Roving response Fourier transforms.

$[A(\omega)] =$ (**n by m**) FRF matrix relating forces to Roving Responses.

$\{F(\omega)\} =$ **m**-vector of (unmeasured) force Fourier transforms.

$\omega =$ frequency variable.

n = number of Roving responses.

m = number of (unknown) forces.

Similarly a MIMO model for the ***Reference*** responses can be written as follows,

$$\{Y(\omega)\} = [B(\omega)]\{F(\omega)\} \qquad (2)$$

where:

$\{Y(\omega)\} =$ **r**- vector of Reference response Fourier transforms.

$[B(\omega)] =$ (**r by m**) FRF matrix for Reference Responses.

r = number of Reference responses.

Equations (1) & (2) both assume that **m** forces are applied to the structure. The number of forces **m**, and the DOFs where they are applied ***assumed to be unknown***.

Multi-Reference Cross Power Spectrum Matrix

A power spectrum matrix can now be formed between the Roving and Reference response Fourier transform vectors. This multi-reference Cross Power spectrum matrix is written as,

$$[G_{x,y}(\omega)] = [A(\omega)][G_{f,f}(\omega)][B(\omega)]^T \qquad (3)$$

where:

$[G_{x,y}(\omega)] = \{X(\omega)\}\{Y(\omega)\}^T$ = (**n by r**) Cross Power spectrum matrix.

$[G_{f,f}(\omega)] = \{F(\omega)\}\{F(\omega)\}^T$ = (**m by m**) force Power spectrum matrix.

T – denotes the transposed complex conjugate.

The force Power spectrum matrix is symmetric and real valued. The Roving & Reference FRF matrices are also assumed to be symmetric.

Each element of the Cross Power spectrum matrix (3) is the Cross Power spectrum between a pair of Roving & Reference responses. Each column of the Cross Power spectrum matrix can be used for OMA after the De-Convolution window is applied to it.

Measurement of elements of the Cross Power spectrum matrix (3) is a straightforward calculation in all current day Fourier analysis systems. A Cross spectrum calculation has the following important advantages,

- Extraneous noise is removed by spectrum averaging.
- Non-linear responses due to random excitation are removed by spectrum averaging.
- Triggering is not required.

To see more clearly how an FRF curve fitting model can be applied to a column of Cross Power spectrum data, and hence OMA can be performed, consider the case of one Reference.

Cross Power Spectrum of One Reference

Using equation (3), the column of Cross Power spectra corresponding to Reference DOF **k** is written,

$$\begin{aligned}
G_{1,k}(\omega) &= \sum_{i=1}^{m}\left[\sum_{j=1}^{m} A_{1,j}(\omega)\, C_{j,i}(\omega)\right]\mathbf{B}_{i,k}(\omega)^* \\
G_{2,k}(\omega) &= \sum_{i=1}^{m}\left[\sum_{j=1}^{m} A_{2,j}(\omega)\, C_{j,i}(\omega)\right]\mathbf{B}_{i,k}(\omega)^* \\
&\vdots \\
G_{n,k}(\omega) &= \sum_{i=1}^{m}\left[\sum_{j=1}^{m} A_{n,j}(\omega)\, C_{j,i}(\omega)\right]\mathbf{B}_{i,k}(\omega)^*
\end{aligned} \qquad (4)$$

where:

$C_{j,i}(\omega) =$ element (j,i) of the force Power Spectrum matrix, $[G_{f,f}(\omega)]$.

* - denotes the complex conjugate.

Equation (4) shows that each component of the k^{th} column of the Cross Power spectrum matrix is a ***summation of terms***, each term taking the form,

(Roving FRF) **x** (force spectrum element) **x** (Reference FRF)*

Flat Force Spectrum Assumption

If the force Spectrum matrix is assumed to be "***relatively flat***" in the frequency range of the resonances of interest, and the excitation forces are ***stationary***, then the Force power spectrum can be replaced with constants for the frequency range(s) of interest.

There are practical cases when the excitation forces can be assumed to have a relatively flat spectrum. For instance, vehicle traffic on a bridge or wind blowing against a building are assumed to be broad-band and random in nature. If the excitation forces are impulsive in nature, they too can be assumed to have a relatively flat spectrum.

With the above assumptions, equation (4) can be re-written as a single summation of terms,

$$\begin{aligned}
G_{1,k}(\omega) &= \sum_{i=1}^{m} D_i A_{1,i}(\omega)\, \mathbf{B}_{i,k}(\omega)^* \\
G_{2,k}(\omega) &= \sum_{i=1}^{m} D_i A_{2,i}(\omega)\, \mathbf{B}_{i,k}(\omega)^* \\
&\vdots \\
G_{n,k}(\omega) &= \sum_{i=1}^{m} D_i A_{n,i}(\omega)\, \mathbf{B}_{i,k}(\omega)^*
\end{aligned} \qquad (5)$$

where:

D_i = constant due to the forces applied at the i^{th} DOF.

The De-Convolution Window

Equation (5) makes it clear that each component of a column of a multi-reference Cross Power spectrum matrix is a summation of terms involving the a ***Roving response FRF multiplied by the complex conjugate of a Reference response FRF***. The inverse Fourier transform of a Cross spectrum is a Cross correlation function. Since multiplication in one domain is equivalent to convolution in the other domain, the inverse Fourier transform of equation (5) is a column of Cross correlations, each column being a summation of Roving response IRFs convolved with a Reference IRF. Figure (2) shows an example Cross correlation function.

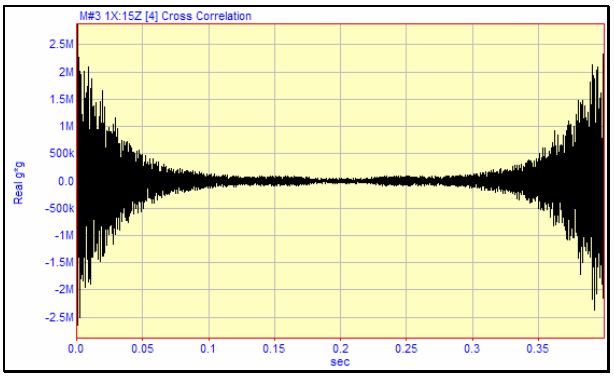

Figure 2. A Cross Correlation Function.

The unique property of these Cross correlations is that the ***Roving IRF dominates the first half of the signal*** while the Reference IRF (corresponding to the complex conjugate of the Reference FRF), dominates the second half of the signal.

To identify the modes of the structure, only the Roving IRF is needed. Hence applying a De-Convolution window to the signal in Figure 2 preserves the Roving IRF is while the majority of the Reference IRF is removed, as shown in Figure 3. Notice that even though the signal is non-zero in the middle of the window, the De-convolution smoothly transitions it to zero.

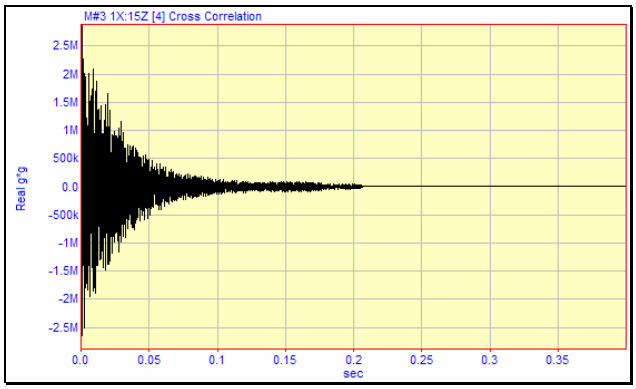

Figure 3. Cross Correlation After De-Convolution Windowing.

When the windowed data in Figure 3 is transformed back to the frequency domain, an FRF curve fitting method can be applied to the Roving FRF to identify its modal parameters.

Illustrative Example

To demonstrate the use of the De-Convolution window, we will start with a modal model of a structure and use equations (1) and (2) to synthesize its responses to a broad band random excitation force. Then a set of Cross Power spectra calculated between multiple Roving and Reference acceleration response will be post-processed using the De-Convolution window and FRF curve fitting. Finally, the resulting modal parameters will be compared with the parameters of the original modal model to provide a ***round trip*** validation of this approach to OMA.

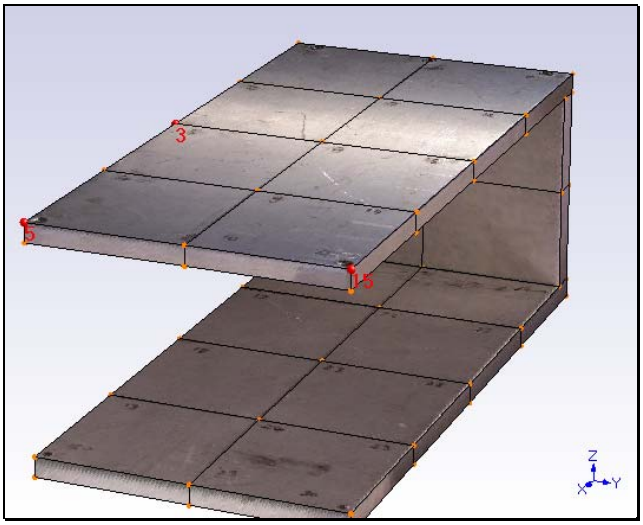

Figure 4. Beam Structure.

The modal model is a set of 10 mode shapes extracted from a set of 99 FRF measurements taken in three directions at 33 points of the structure shown in Figure 4.

An FRF matrix model was synthesized using the modal model of the beam, and the FRF model was ***"excited"*** with a burst random signal at the ***left front corner*** of the upper plate, at DOF 5Z. Several typical structural responses to the bust random excitation are shown in Figure 5.

Burst Random Excitation for Leakage Free Responses

Of course, burst random excitation of a real structure could never occur unless the structure was artificially excited with shakers driven by burst random signals. In most practical cases where operating data is acquired, the responses would be purely random, and therefore a Hanning window would be applied to the response signals in order to ***reduce leakage*** in the Cross spectrum measurements. By using burst random excitation, our MIMO simulation provided acceleration responses that would yield ***leakage-free*** Cross Power spectrum estimates, thus eliminating leakage as a source of error in this round trip experiment.

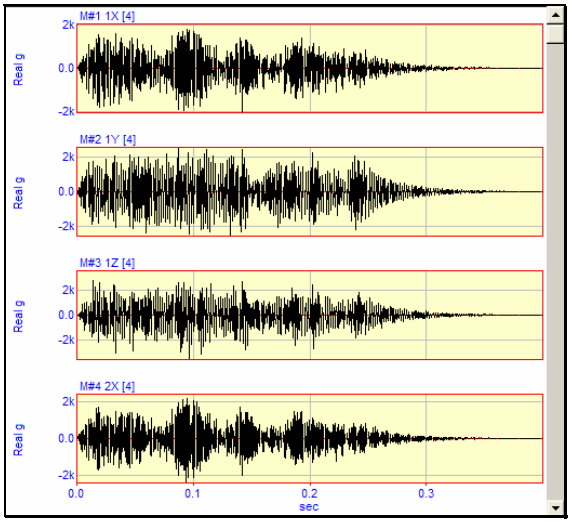

Figure 5. Several Acceleration Responses.

As a first simulated OMA, the acceleration response at the ***right front corner*** of the upper plate was used as a reference response, and Cross Power spectra were calculated between all 99 DOFs and the reference DOF 15Z.

Some typical Cross spectra are shown in Figure 6A before the De-convolution window was applied. These measurements are the results of 50 spectrum averages, with 1024 lines of resolution.

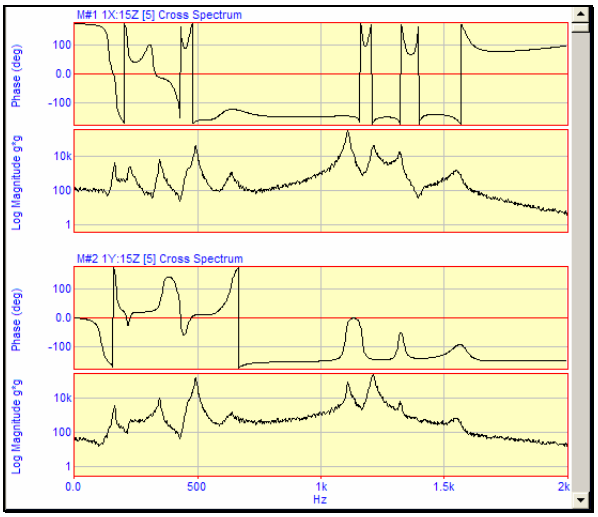

Figure 6A. Cross Spectra before De-Convolution Windowing.

The same Cross spectra are shown in Figure 6B after the De-convolution window was applied. Notice that the magnitudes of the Cross spectra have less noise on them due to the effect of the De-Convolution window. Notice also that the phases now have the negative changing phase as frequency increases through a resonance peak.

The windowed Cross spectra where curve fit using an FRF curve fitter, and the results are shown in Table 1.

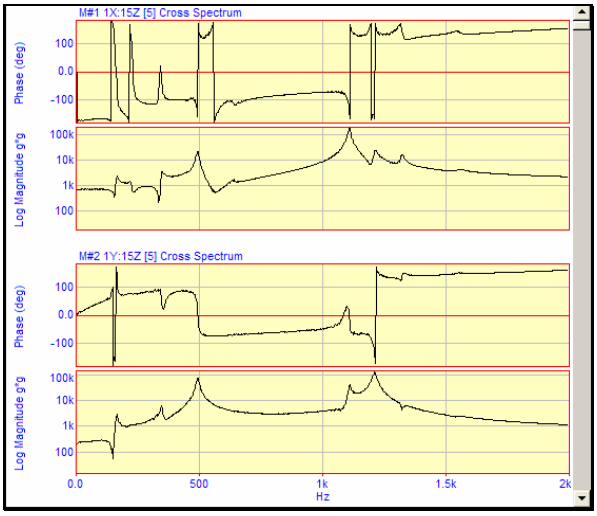

Figure 6B. Cross Spectra after De-Convolution Windowing.

	Frequency (Hz)		Damping (Hz)		MAC (%)
Mode	**Modal Model**	**Curve Fit**	**Modal Model**	**Curve Fit**	**Reference 15Z**
1	164.6	164.8	3.07	3.23	100
2	224.1	224.9	6.61	5.70	99.3
3	347.4	346.2	5.22	4.86	100
4	460.7	460.1	10.91	10.14	**16.9**
5	492.8	491.9	4.61	5.12	100
6	634.3	630.1	14.19	8.57	97.5
7	1108.1	1108.8	5.07	4.78	100
8	1210.1	1209.5	7.21	6.83	99.9
9	1322.3	1322.1	7.12	7.10	99.7
10	1553.9	1554.7	19.57	18.03	99.3

Table 1. Operating Modes from Reference 15Z.

Table 1 indicates that all of the modal frequencies and damping were recovered with reasonable accuracy from the Cross spectra, but it's clear from its low Modal Assurance Criterion (MAC) value that the 460 Hz mode shape was not recovered. It can only be assumed that the 460 Hz mode was not well defined in set of Cross Power spectra using Reference DOF 15Z, which "***implicitly multiplied together***" the Roving and Reference FRFs between the excitation DOF 5Z and the reference DOF 15Z.

Multiple Reference OMA

Since the Reference DOF 15Z was not enough to correctly identify all of the modes in the modal model, a second simu-

lated OMA was performed using two Reference responses (3Z & 15Z). Table 2 below shows the results of the multiple reference OMA. The results clearly indicate that the extra reference not only provided better estimates of modal frequency and damping, but ***more accurate mode shape estimates from both references*** as well.

	Frequency (Hz)		Damping (Hz)		MAC (%)	MAC (%)
Mode	Modal Model	Curve Fit	Modal Model	Curve Fit	Ref. 3Z	Ref. 15Z
1	164.6	164.4	3.07	3.01	99.9	100
2	224.1	224.1	6.61	5.84	99.8	99.3
3	347.4	347.3	5.22	5.43	100	100
4	460.7	461.8	10.91	9.94	98.8	**95.8**
5	492.8	492.6	4.61	4.16	100	100
6	634.3	635.5	14.19	11.01	99.7	99.3
7	1108.1	1108.3	5.07	4.73	100	100
8	1210.1	1209.1	7.21	7.06	100	100
9	1322.3	1321.9	7.12	6.60	99.9	99.9
10	1553.9	1551.9	19.57	17.98	99.3	99.6

Table 2. Operating Modes from References 3Z & 15Z.

A typical curve fitting result is shown in Figure 7, where the red line is the fit function plotted on top of the Cross spectrum data. These results indicate that an FRF curve fitter can be applied quite successfully to operating data after a De-Convolution window has been applied to it.

CONCLUSIONS

It was demonstrated the OMA can be performed quite accurately using response only (or output only) accelerometer data from a structure, it the excitation forces have a ***relatively flat spectrum*** over the frequency range of interest, and the data is processed with a De-Convolution window. It was shown in the background theory section that the De-Convolution window removes the majority of the Impulse Response Function (IRF) of the Reference response from the Cross correlation function, leaving the IRF of the Roving response. Transforming the Roving IRF to the frequency domain yields an FRF which can be processed using FRF curve fitting methods to extract modal parameters.

Another advantage of this approach is that after applying the De-Convolution window, more zero valued samples can be added to the IRFs to increase their time length. Then, when the IRFs are transformed into FRFs, the frequency resolution of the FRFs in increased, thus providing more samples of data surrounding each resonance peak, which improves the curve fitting results.

This combination of De-Convolution windowing and increased frequency resolution is a powerful combination of techniques for performing an OMA.

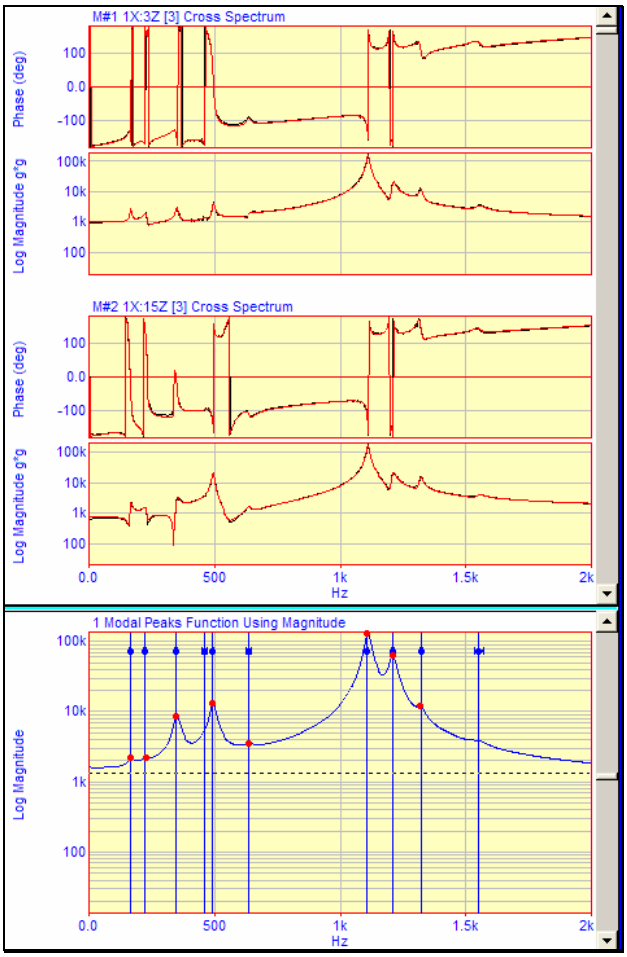

Figure 7. Typical Cross Spectrum Curve Fitting Result.

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