

## Modal Analysis Versus Finite-Element Analysis

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This month's issue is about computer-aided engineering, or CAE. Most engineering work today relies heavily on the use of digital computers, and most engineering organizations employ a variety of CAE tools to develop new products, increase the performance of existing products and troubleshoot problems that occur in the field. Modal analysis and finite-element analysis are two popular CAE tools.

The terms in the title of this editorial require a brief explanation. Modal analysis has become the favorite label for what is more accurately called experimental modal analysis (EMA), or modal testing, or from the old days, a modal survey. EMA is the activity of an experimentalist who endeavors to characterize the dynamic behavior of a structure in terms of its modes of vibration.

Finite-element analysis (FEA) is also done to model structural dynamics, but with a computer program instead of by experiment. FEA is the activity of a structural analyst and can also provide the modes of a structure. FEA is analytical, EMA is experimental, and modes are the common ground between the two.

The word 'versus' in the title has two meanings, both of which apply in this case. First, it means, "contrasted with" or "compared to." But it also has a second meaning according to my dictionary. In "law and sports," it means 'against' or "opposed to." Now, I don't believe that either EMA or FEA has much to do with law or sports, but I've experienced the effects of the second meaning on several occasions during my career as an experimental practitioner.

In fact, these two engineering tools are complimentary. If an EMA and an FEA on the same structure both yield the same modes, then presumably both must be accurately characterizing its structural dynamics. One should not be used to the exclusion of the other. Both tools are useful for gaining a better understanding of the dynamic behavior of physical structures, and in particular for simulating and ultimately solving resonant vibration problems.

FEA has grown steadily in popularity as an engineering tool since the 1960s. In 1965, the NASTRAN program was developed by NASA to support the engineering development of space vehicles. Since then, many companies have improved upon and commercialized NASTRAN, and many other FEA programs have been commercialized as well.

In the early days when EMA was called

a modal survey, it was done primarily to validate the accuracy of an FEA model. Modal surveys used multiple shakers driven with sinusoidal signals and attempted to excite structures one mode at a time. These systems were usually large and expensive, and a complete modal survey was time consuming.

Today, EMA is done much differently. Starting in the 1970s, the development of lower cost multichannel data acquisition systems, the use of the FFT (fast-Fourier transform) algorithm, and digital signal processing has fostered the widespread use of broadband structural testing. Broadband excitation is now done with an impact hammer or with shakers driven by random signals. Many modes are excited simultaneously, and further processing is used to extract their parameters from the data. Both the cost and the time required to perform an EMA have been drastically reduced with this new approach.

EMA is still used to validate FEA models, but it is also heavily used for troubleshooting noise and vibration problems in the field. Once an FEA model has been validated, it can be used for a variety of static and dynamic load simulations. An equivalent model in the form of modal parameters, called a modal model, can also be used for simulations.

The most common simulation is to apply anticipated static loads to a model to see whether or not the structure is strong enough to support the loads without breaking. For static load simulation, only the stiffness properties of the structure are required. These are represented in the model with a stiffness matrix, which is generated by the FEA program. For a dynamic simulation, the inertia properties are also required, which are modeled with a mass matrix.

When an FEA model contains both mass and stiffness, the differential equations of motion defined by these matrices can be solved for the modes of the structure. Modes describe resonances. If resonances are excited, they can be very damaging to a structure, which is why we often want to know something about them.

Each mode is defined by three different kinds of parameters: its modal (or resonant) frequency, its modal damping, and its mode shape. Each mode is directly influenced by the mass and stiffness of the structure and its boundary conditions (more on these later). Mass and stiffness are calculated from the geometry and the properties of the materi-

als, density, elasticity (strength), and Poisson's ratio (squeeze effect), out of which the structure is made.

But what about damping? Certainly, every structure that vibrates will eventually stop vibrating if all of the forces causing it to vibrate are removed. Vibration will stop because some kind of damping mechanism, or in most cases a combination of mechanisms, will dissipate the energy from the structure to its surroundings. In the earth's atmosphere, one damping mechanism that is always present is the surrounding air. All structures that vibrate in the atmosphere are acting like stereo speakers; that is, they are pushing air with their surfaces as they vibrate.

This type of damping is primarily a viscous mechanism, similar to the way an automotive shock absorber behaves. Viscous damping is modeled as a constant multiplied by velocity. In the case of a structure moving the surrounding air, the velocity would be the velocity of the surface of the structure. This is all fine and good, but how can the viscous damping of the surrounding air be modeled, let alone the other damping mechanisms that might be at work in a structure? The answer is, damping can't be easily modeled, so it's not included in most FEA models. But modes can still be obtained from a model with no damping in it. Inertia and stiffness cause resonant vibration. Damping only dissipates it.

Unlike modeling, an EMA is always done on a structure that has damping in or around it. Damping is unavoidable when testing a real structure. So, how can experimental modes, which include the effects of damping, be compared with analytical modes that don't? The quick answer, for those of you who aren't worried about the details, is that they can be compared. Most structures, especially those with troublesome resonances, are "lightly damped." Consequently, experimental mode shapes will closely approximate analytical shapes.

Moreover, resonances usually cause large amplitudes of response, which show up as peaks in the frequency spectrum of a response measurement. By means of some kind of curve fitting process (there are many), EMA can provide accurate estimates of modal frequency and damping from response spectrum measurements.

There is usually a large disparity between the sizes of experimental and analytical mode shapes, but there are ways of dealing with it. A typical EMA may

include hundreds of measurements, which will yield experimental mode shapes with hundreds of components or DOFs (degrees-of-freedom). A mode-shape DOF is the deflection of the structure at a point in a direction. On the other hand, an FEA typically yields analytical mode shapes with thousands or even millions of DOFs in them. Each analytical shape has a modal frequency associated with it but no modal damping.

Although experimental shapes will have far fewer DOFs than analytical shapes, both shapes should be comparable at all DOFs that are common between them. Two different methods have become popular for quantitatively comparing mode shapes – the orthogonality check and the modal assurance criterion, or MAC.

The orthogonality check was first adopted in the early days when EMA was done to confirm FEA models. An orthogonality check attempts to ‘diagonalize’ the mass matrix of the FEA model, using both the experimental and analytical mode shapes. There are more details required to completely understand this process, including the nontrivial step of reducing the size of the mass matrix to match the experimental mode shapes. Nevertheless, the end result is this: if the experimental mode shapes are the same as the analytical mode shapes, then the orthogonality check will yield a diagonal matrix, also

called the modal mass matrix.

On the other hand, the MAC method only requires the mode shapes themselves. By eliminating the mass matrix from the calculation, one possible source of error is removed from the shape comparison. MAC also yields a diagonal matrix when the experimental and analytical shapes are the same.

When either the orthogonality check or MAC yields a diagonal matrix, then both sides (the experimentalists and analysts) are happy. When either check fails to yield a diagonal matrix, then the finger pointing begins. This is where I’ve experienced the second meaning of ‘versus.’ Which mode shapes are correct? Of course, each side believes in its own results.

There are a number of reasons why experimental mode shapes don’t match analytical shapes. A significant one is that the boundary conditions may be different between the EMA and FEA. If the boundary conditions are different, the mode shapes will be different. For example, the modes of a cantilever beam are clearly different from those of a free-free beam. It is often difficult to reproduce in an EMA the same boundary conditions that were used during construction of the FEA model. Conversely, the flexibility of floors, walls, platforms, mounts, and all types of boundaries that may be assumed as rigid in an FEA, may significantly af-

fect the modes of the real structure. This is where the complementary nature of these two tools becomes important.

The structural dynamicist must ask why the results are different. Is there a substantial modeling error, or is the experimental error significant? Having both tools available gives the dynamicist the advantage of verifying modal frequencies and measuring modal damping with EMA and defining mode shapes with a large number of DOFs from FEA. The ideal modal model might be one that combines experimental frequencies and damping with analytical mode shapes once the experimental and analytical shapes have been matched at common DOFs.

In the past, many engineering organizations have separated EMA from FEA, have located experimentalists and analysts in different departments and buildings, and have not supported cooperation between the two groups. Fortunately, this is changing. Many organizations are now starting to benefit from the convergence of EMA and FEA. Since they are complementary, it makes sense to take advantage of the strengths of each tool.

Structural dynamicists of the future will be better equipped to understand and solve structural noise and vibration problems when they have both EMA and FEA in their toolboxes. 

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