PARAMETER ESTIMATION FROM FREQUENCY RESPONSE MEASUREMENTS USING RATIONAL FRACTION POLYNOMIALS (TWENTY YEARS OF PROGRESS)

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ABSTRACT

This paper was originally presented at the first IMAC Conference in 1982 [6]. Its abstract was a follows,

This is a new formulation that overcomes many of the numerical analysis problems associated with an old least squared error parameter estimation technique. Overcoming these problems has made this technique feasible for implementation on mini-computer based measurement systems.

This technique is not only useful in modal analysis applications for identifying the modal parameters of structures, but it can also be used for identifying poles, zeros and resonances of combined electro-mechanical servo-systems.

Our early development of this method took place in the late 1970's, and it was first implemented (as an SDOF fitter) in a modal testing instrument, the Hewlett Packard 5423A Structural Dynamics Analyzer. Since then it has been used in a variety of other analyzers and modal analysis software packages (as an MDOF & Global fitter), and is still widely used today.

In this paper, we will review other IMAC papers that have been written about this method during the past twenty years. We will also discuss remaining curve fitting challenges and opportunities for further research.

INTRODUCTION

We begin by reiterating the main points of our original paper. Excerpts from it are *italicized*.

This paper presents the results of an algorithm development effort that was begun back in 1976. At that time, we were looking for a better method for doing curve fitting in a mini-computer based modal analysis system. This type of system is used to make a series of FRF measurements on a structure, and then perform curve fitting on these measurements to identify the damped natural frequencies, damping, and mode shapes of the predominant modes of vibration of the structure.

The three main requirements for a good curve fitting algorithm in a measurement system are 1) execution speed, 2) numerical stability, and 3) ease of use.

Complex Exponential

A well-known curve fitting algorithm (called the Complex Exponential) had already undergone much development by the mid 1970's, and was to remain the basis for many further developments as a time-domain method. We too had experimented with the Complex Exponential, and made the following comments about it,

Previous to this development effort, we had experimented with a well-known curve fitting algorithm called the Complex Exponential, or Prony algorithm. This algorithm has undergone a lot of refinement ([2], [3]) and is computationally very efficient and numerically stable in 16-bit machines. However, it curve fits the impulse response function instead of the FRF. The impulse response can be obtained by taking the Inverse Fourier transform of the FRF. When the FFT is used to obtain the impulse response from an FRF measurement, a potentially serious error can occur, which is called wrap around error, or time domain leakage. This error is caused by the truncated form (i.e. limited frequency range) of the FRF measurement, and distorts the impulse response as shown in Figure 1.

Hence, we sought to develop an algorithm with some of the same characteristics as the complex exponential method, (e.g., it is easy to use along with being numerically stable), but that curve fits the FRF measurement data directly in the frequency domain.

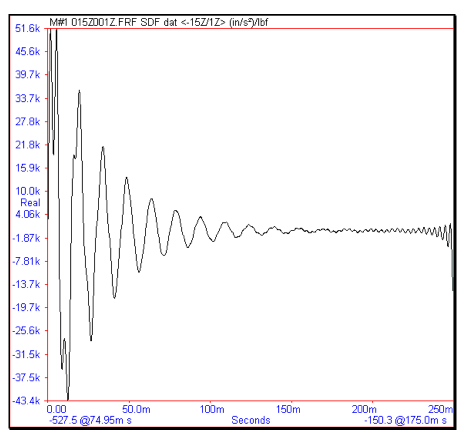


FIGURE 1. Impulse Response with Leakage.

Time Domain Polyreference Method

In the same year of our original paper 1982, another time domain curve fitting method called Time Domain Polyreference, was introduced. It is probably the most widely used curve fitting method of all [11]. This method, which is an extension of the Complex Exponential method, can be applied to a multiple reference set of Impulse Response Functions. More technical papers have been written about it than any other method.

Ibrahim Time Domain Method

Another very popular time domain curve fitting method was introduced in 1977 [9], several years prior to our original paper. It became know as the Ibrahim Time Domain method. This method has been widely used by the aerospace community.

Iterative Frequency Domain Curve Fitting

Prior to the RFP development, we had also gained some experience with an *iterative* frequency domain curve fitting algorithm, described in [1] and implemented in a commercially available modal testing system (Option 402 to the Hewlett Packard 5451B Fourier Analyzer System). This package was first sold in 1974.

The difficulty with any iterative technique is that it may not converge on a usable solution. The attraction of the RFP algorithm was that is was not iterative,

If it is assumed that the frequency response measurement is taken from a linear, second order dynamical system, then the measurement can be represented as a ratio of two polynomials, as shown in Figure 2.

Rational Fraction Form

$$\mathbf{H}(\omega) = \left. \frac{\sum_{k=0}^{m} a_k s^k}{\sum_{k=0}^{n} b_k s^k} \right|_{s=j\omega} \qquad (1)$$

Partial Fraction Form

$$\mathbf{H}(\omega) = \sum_{k=1}^{n/2} \left[\frac{r_k}{s-p_k} + \frac{r_k^*}{s-p_k^*} \right]_{s=j\omega} \qquad (2)$$

$$p_k = -\sigma_k + j\omega_k = k^{th} \text{ pole}$$

$$r_k = \text{residue for } k^{th} \text{ pole}$$

FIGURE 2. Analytical Forms of the FRF.

In the process of curve fitting this analytical form to the measurement data, the unknown coefficients of both the numerator and denominator polynomials, (a_k, $k=0,\ldots,m$) and (b_k, $k=0,\ldots,n$), are determined. It is shown later that this curve fitting can be done in a least squared error sense by solving a set of linear equations, for the coefficients.

Ill Conditioned Equations

Our original introduction to the RFP curve fitting method came from reference [5]. In it, the authors presented the solution equations, but concluded that they were ill conditioned and could not be solved by computer, even for simple cases. We also verified that this was indeed the case in our original implementation of the algorithm.

Orthogonal Polynomials

A major breakthrough occurred when we reformulated the curve fitting problem in terms of orthogonal polynomials, as prescribed by Forsythe in reference [5]. This straightforward application of Forsythe polynomials, together with further computational efficiencies due to the symmetrical properties of the FRF, yielded solution equations that could easily be solved in a mini-computer. These efficiencies were presented in [6].

But most of all, the solution equations using orthogonal polynomials were numerically stable, exhibiting a dramatic improvement over the use of ordinary polynomials. Furthermore, the problem size was essentially “cut in half” by the use of orthogonal polynomials, which uncoupled the solution equations for the denominator polynomial coefficients from those for the numerator coefficients.

When exciting modes of vibration in a structure, or in making measurements in a servo-loop, the denominators of all the measurements should contain the same characteristic polynomial. In the case of structural resonances, this is equivalent to saying that modal frequencies and damping are the same, no matter where they are measured on the structure. Alternatively, the poles of a servo-loop can be identified from measurements between any two points in the loop.

As pointed out earlier, one of the advantages of formulating the solution equations in terms of orthogonal polynomials is that the unknown characteristic polynomial coefficients $\{D\}$ *can be determined independently of the numerator coefficients* $\{C\}$.

Global Curve Fitting

Having uncoupled the solution equations through the use of orthogonal polynomials, it was clear that a two-step global curve fitting approach could be implemented. First, the coefficients of the denominator (or characteristic) polynomial are estimated by curve fitting *multiple* FRF measurements.

Since ideally, all FRFs measured from the same structure should have the same denominator, a better estimate of the characteristic polynomial can be obtained by curve fitting all of the measured FRFs. The roots of this polynomial are the modal frequencies & damping, which are global properties of the structure.

Then, a second curve fitting step is performed, where the numerator polynomial coefficients are estimated by curve fitting *each* FRF by itself. The numerator polynomial coefficients are then used in a numerical partial fraction expansion to yield the residue (mode shape component) for each mode and each FRF.

In references [7] (1985) & [8] (1986), we presented all of the details of these two steps of global curve fitting using the RFP method.

Compensation for Out-of-Band Resonances

Another important issue with the use of any curve fitter is how to compensate the residual effects of out-of-band modes. Out-of-band effects can "contaminate" the FRF data it two ways, as explained below,

FRF measurements are always made over a limited frequency range by exciting the structure or system with some broad band signal. As a consequence, the measurements will typically contain the residual effects of resonances, which lie outside of the measurement frequency range. In addition, we normally curve fit the measurement data only in a more limited frequency range surrounding the resonance peaks. Hence, to give accurate results, all curve fitters must somehow compensate for the residual effects of resonances, which lie outside of the curve fitting frequency range.

Regardless of whether a curve fitting method uses time domain or frequency domain data, the residual effects of out-of-band modes must be dealt with. The RFP method offered a unique advantage that was not available with the Complex Exponential method, for instance.

With this curve fitter, out-of-band effects can be approximated by specifying additional terms for either the numerator or the denominator polynomial.

In the original paper [1], we explored the uses of both extra numerator and extra denominator terms to compensate for out-of-band effects. Adding extra denominator terms is the same as adding extra modes to the curve fitting model or solution equations.

The Complex Exponential method almost always requires the use of extra modes in its curve fitting equations in order to obtain valid results. The difficulty with using extra modes in the model is that the results must then be sorted out, and the "good" modes (with valid parameter estimates) separated from the "computational" modes (with invalid estimates). The frequency & damping Stability diagram was developed for this purpose. A Stability diagram is essentially a listing of frequency & damping estimates, for a single mode model, then two modes, three modes, and so on. The user must then pick valid parameter estimates from the diagram.

Extra Numerator Terms

We discovered however, that the addition of extra numerator polynomial terms was a more practical way to account for the residual effects of out-of-band modes. These extra terms have commonly been referred to as "**inertial restraint**" and "**residual flexibility**".

The use of these extra terms does not increase the order of the characteristic polynomial. Hence, there are no extra "computational" modes in the results. Furthermore, the extra numerator polynomial terms are merely "thrown away" by the numerical partial fraction expansion; so valid residue estimates for the modes in the model can be obtained.

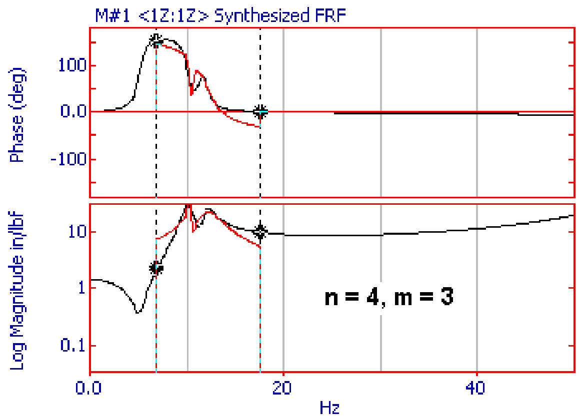
In the original paper, this point was illustrated with an example that included two closely coupled modes (at 10 & 12 Hz) and a third out-of-band mode (at 60 Hz). We concluded,

It is clear that the parameters of the first two modes cannot be identified without some form of compensation for the third mode. Again, we can attempt to compensate for the effects of the third out-of-band mode by adding more terms to the numerator polynomial. Figure 3 also shows the results of this curve fit with ***n=4***, ***m=7***. *In this case, the additional numerator terms do an excellent job of compensating for the third mode.*

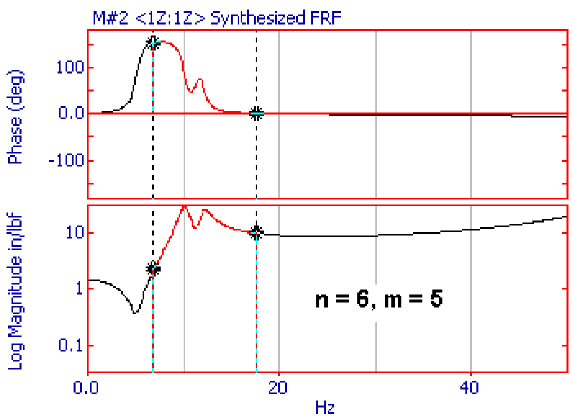
Finally, we can also compensate for the third out-of-band made by adding another DOF to the denominator. Figure 3 shows the results of the curve fit with ***n=6***, ***m=5***. *Notice that this fit function is, as expected, a perfect match to the idealized FRF measurement, and that even the 60 Hz mode which lies far outside the curve fitting band is also correctly identified.*

Even though for this simple case, adding a third mode to the model worked equally as well as adding extra numerator polynomial terms, in general one would not know how many modes lie outside the curve fitting band. Furthermore, there is not control in the RFP method over where the mode frequency estimates will lie. Computational modes can lie inside the curve fitting band and make it difficult to decide whether or not they are computational without the aid of a Stability diagram or some other method.

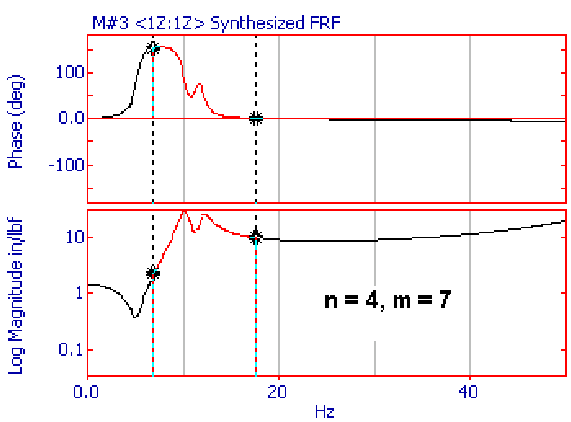
In general, we have found that the over-specification of the numerator polynomial order (using from 2 to 8 extra terms) is a much better way to compensate for out-of-band effects than using extra computational modes.



Mode	Frequency (Hz)	Damping (%)	Res Mag (in/lbf-sec)	Res Phs (deg)
1	10.26	1.261	55.95	132.8
2	12.07	10.13	410.7	148



Mode	Frequency (Hz)	Damping (%)	Res Mag (in/lbf-sec)	Res Phs (deg)
1	10	5	200	180
2	12	4.5	150	180
3	59.9	2.088	2.495E3	359.5



Mode	Frequency (Hz)	Damping (%)	Res Mag (in/lbf-sec)	Res Phs (deg)
1	9.999	4.996	199.8	180.1
2	12	4.499	149.9	179.9

FIGURE 3. Out-Of-Band Compensation.

FURTHER WORK WITH THE RFP METHOD

We surveyed the past 19 years of IMAC proceedings for other papers that referenced the RFP method. We found a total of 33 papers in the IMAC proceedings. (We have also included a Sound & Vibration magazine article that addressed numerical issues related to the RFP.)

We acknowledge that other technical journals & conference proceedings could contain additional information about the RFP method. Due to time constraints however, we limited our search to the IMAC proceedings.

The papers we surveyed are organized by topic and year of publication in the References section.

Some papers have extended its usefulness by introduced improvements to the method. Some papers merely referred the RFP as a modal parameter estimation method. Some compared the RFP with other curve fitting methods in survey papers.

Extending the RFP Method

We found 21 papers that proposed ways to extend the usefulness of the RPF method. Following is a brief review of the improvements we found in some of these papers.

Chebycheff Polynomials

Several papers [14], [16] & [24] used Chebycheff polynomials instead of the Forsythe polynomials that we used. These authors found better computational efficiency and improved numerical accuracy with the Chebycheff polynomials.

Reference [24] also found that solving for the roots of the characteristic equation in terms of orthogonal polynomials permitted solutions of much higher order characteristic polynomials. Solutions of more than 10 modes at a time were achievable. This is difficult if not possible when the orthogonal are converted to ordinary polynomials before root solving,

Non-Uniform Frequency Axis

Several authors [14], [19] & [23] also pointed out that the RFP method can be formulated, and works equally well with non-uniformly space frequency axis data, such as log frequency data. Since the RFP works best using small frequency bands of FRF data surrounding resonance peaks, its use on log axis data is a logical extension of this capability.

Weighting Functions

Several authors [14], [22] & [23] discussed the influence of bias errors in the FRF data and its effect on the accuracy of modal parameter estimates. One paper [14] proposed using the Coherence function to construct weighting functions. This emphasizes data surrounding resonance peaks, and ignores data where the Coherence values are low.

The authors of [22] implemented the RFP using a non-linear least squared error function. This results in a set of non-linear solution equations that must be solved using iterative techniques, but can remove the effects of bias errors.

Global Curve Fitting

Several authors [21], [23], [26] discussed the use of the RFP for global curve fitting of multiple FRFs to obtain frequency & damping estimates. Although we too showed how to perform global fitting in our original paper by taking advantage of the uncoupled solution equations, we didn't give the details until our later 1985 & 1986 papers [7] & [8].

Multiple Reference Curve Fitting

Several papers [18], [19], [20] have extended the RFP to curve fitting of a multiple reference set of FRFs. Multiple reference curve fitting was made popular by the time domain Polyreference method, discussed earlier. A multiple reference set of FRFs is required in order to correctly identify closely coupled modes and repeated roots.

Two or more modes are closely coupled if they are represented by only one resonance peak in the FRFs. Repeated roots are two or more modes with the same modal frequency, but different mode shapes.

Mode Indicator Functions

Some authors [29], [31] introduced the idea of using the RFP together with a mode indicator function. Two types of mode indicators are popular; The Multivariate Mode Indicator (MMIF), and the Complex Mode Indicator function (CMIF).

These mode indicator functions are useful for two reasons;

1. They indicate how many modes are present in a frequency band, thus providing an estimate of the correct curve fitting model size.
2. They provide modal participation factors that are used to weight a multiple reference set of FRFs during curve fitting.

Both of these mode indicators have been effectively used with the RFP for finding the modal parameters of closely coupled modes and repeated roots [12], [13].

CURVE FITTING SURVEY PAPERS

We found 7 papers in the IMAC proceeding that surveyed curve fitting methods. These surveys cover a time span from 1985 to 1997. All of them reference the RFP method. It is also referred to as the OP (orthogonal polynomial) or the MRF (modified rational fraction) method in these and other papers.

Generally speaking, all MDOF (multiple mode) curve fitting methods can be classified as either time domain, based on the Complex Exponential, or frequency domain, based on either the partial fraction form or the rational fraction form of the FRF (*See* Figure 2). Following this distinction, the solution algorithms are either linear, providing one solution, or non-linear and iterative, providing a series of improving solutions.

Global curve fitting methods divide the curve fitting process into two steps, obtaining modal frequency & damping estimates in the first curve fitting step, followed by modal residue estimates in a second step.

Multiple reference (or Polyreference) methods extend global curve fitting by utilizing modal participation factors to weight each reference of FRF data so that the parameters of each mode are estimated using the reference where it is most strongly defined.

GOOD MEASUREMENTS – THE KEY TO CURVE FITTING SUCCESS

The main topic of this article has been applications of and improvements to the RFP curve fitting technique over the last 20 years. However the key to success using this or any other curve fitting technique lies in the quality of the experimental data derived from the test structure. The age old expression "garbage-in-garbage-out", is truly applicable when curve fitting FRF measurements.

During the development of the RFP method, its first test was to determine how well it worked on FRFs that were generated analytically, in other words "perfect" measurements. Obviously, if the algorithm under development cannot produce accurate results using a "perfect" FRF, there would be no need to use it on measurements from a real structure.

We are still amazed how accurate the RFP method is when used on analytically generated FRFs. Even when used on a small frequency range of the FRF data, the RFP still yields extremely accurate modal parameter estimates. This is generally true for most curve fitting methods used in commercially available Modal Analysis systems today.

However, FRF measurements from real structures are usually far from "perfect" FRFs, and here in lies the difficulty with the curve fitting process. There is a multitude of reasons why we don't or can't make accurate FRF measurements that will easily yield meaningful modal parameters.

Measurement problems start with the assumptions, which the curve fitting algorithms are based on, regarding the dynamic behavior of the test structure. Measurement problems are further compounded when non-perfect transducers are used to measure the two signals used to calculate FRFs, namely the excitation force (input) and response motion (output) signals.

Furthermore, the type of excitation used to measure FRFs and how the test structure is mounted can many times affect the quality of the measurements. Finally, one of the major influences on an accurate FRF measurement is the measurement setup of the data acquisition system or spectrum analyzer.

Structural Dynamics Assumptions

All curve fitting algorithms are based upon a mathematical model for the dynamics of the test structure. An analytical formula for an FRF is developed and used to derive the curve fitting algorithm.

Typical assumptions that are made about the dynamics of the test structure are:

- Linear, 2nd Order Differential Equations of Motion
- Reciprocity
- Time Invariance

Structures or systems that satisfy the above assumptions can yield FRF measurements that can be curve fit using most curve fitting algorithms. Any violations of the above assumptions will lead to curve fitting difficulties and errors in the resulting modal parameters.

Linearity

Perhaps the linearity assumption is violated most often when testing real structures. The most common type of non-linearity (i.e. the structure is not linear) is associated with a structure that has hardening or softening stiffness (spring) mechanisms. The response of these structures depends on the excitation level used to measure the FRFs.

Non-linear structures are problematic because they don't exhibit classical modes of vibration. ***Modes of vibration are only defined for linear structures.*** From a structural dynamics point of view, a non-linear structure can be thought of as a family of piecewise linear systems, with a different linear system for each RMS excitation level.

Varying Force Levels

When testing non-linear structures, the excitation technique used is critical for measuring repeatable and pseudo-linear FRF measurements. Because of the lack of control of the force level during the measurement process, impact (hammer) testing is one of the worst excitation techniques for measuring the FRFs of non-linear structures.

When the excitation level is difficult to control during the averaging process (or from one measurement to the next), FRF measurements made on a non-linear structure may change with the excitation level. Consequently, curve-fitting techniques cannot be applied to a set of FRFs taken from a non-linear structure using varying force levels. Many other types of non-linearity's can exist in structures, all of which will cause difficulties when applying curve-fitting techniques that assume the FRFs are from a linear structure.

Testing For Non-Linearity

Several different tests can be used to determine whether or not a structure has non-linear behavior. One straightforward test is to measure the Coherence function (γ^2) along with the FRF measurement. The Coherence is a direct measure of how linearly related the excitation is to the structural response.

When making the first few averages during an impact test, use a small amount of force. Then on the last average, increase the force level. If the structure exhibits a load dependent non-linearity, the Coherence will dramatically decrease in value in the frequency ranges where non-linear behavior occurs.

2nd Order Equations & Reciprocity

Most modal testing assumes that the dynamics of the structure can be modeled using 2nd order differential equations with symmetrical mass, damping, & stiffness matrices. The reciprocity assumption also follows from the symmetry of the matrices.

An FRF is measured between an excitation DOF (point & direction) and a response DOF. Reciprocity also means that an FRF measured between an excitation DOF A and a response DOF B is identical to the FRF measured between excitation DOF B and response DOF A.

Time Invariance

Finally, the assumption of time invariant behavior can many times be easily controlled. For example, if the structure under test changes dynamically with temperature or humidity, these parameters can often be controlled during a test.

Time variance can be a bigger problem for structures that change over time due to mass changes, such as a missile in flight where the mass changes as fuel is expended. Time varying structures have to be treated as a family of structures during testing, each one having its own steady state dynamical properties.

Transducers

FRF measurements are made using a wide variety of different transducer types. The excitation of a structure is generally measured with a load cell that measures force. The response due to the excitation can be measured using either an acceleration, velocity or displacement transducer, or a combination of motion transducers.

All of these transducers convert a physical quantity (force, acceleration, velocity or displacement) to a voltage that is measured with a data acquisition (front end) system. Unfortunately, no transducer is perfect. All transducers have limitations and create errors, which are influenced by the following factors:

- Linearity
- Flatness of Frequency Response
- Sensitivity
- Dynamic Range
- Transverse Sensitivity

Using transducers that are inappropriate can result in errors in the FRF measurements. These errors in turn cause curve fitting difficulties, which usually cause errors in the final results, the modal parameters estimates [52], [53], [54].

Excitation & Response

In order to calculate FRFs, the structure must be excited with a measurable force. The type of excitation depends on the type of structure being tested. If a structure is linear, the type of excitation is not critical.

However, when testing a non-linear structure, the excitation method can dramatically affect the FRF measurements. The worst method for exciting a ***load non-linear*** structure is impacting it, as previously discussed. Shaker excitation using a random or burst random signal is better suited for testing this type of non-linear structure.

The locations of the excitation & response measurement DOFs are directly related to the dynamical properties (modal parameters) of a structure. Excitation close to a mode shape nodal point will result in a lower level response for the mode in question *in all FRFs*. Conversely, if a (fixed) reference response is used, transducer placement close to mode shape nodal point will also result in a lower response level for the mode in question *in all FRFs*, [55], [56], [57], [58].

Analyzer Setup

One of the biggest sources of error in measuring FRFs is improper or sub-optimal analyzer setup. Selection of the appropriate signal-processing windows is a common source of error. Signal processing windows are either overlooked altogether, or the wrong choice is made. For example, the common mistake of using a Hanning window on both the excitation and response signals during an impact test results in extremely distorted FRF measurements.

Leakage

Signal processing windows are used to minimize the leakage effects that occur when processing signals that don't meet the assumptions of the FFT algorithm. The following types of signals meet these assumptions:

- Periodic in the sampling window
- Completely contained in the sampling window

If your test signals are not periodic or completely contained in the sampling window (the time domain record of samples), then leakage errors will occur. These errors can affect both the frequency and amplitude of the spectrum.

Most FFT analyzers contain signal processing windows that are designed to minimize the effects of leakage. Even when the appropriate window is chosen however, the resulting FRFs will still contain some amount of error due to leakage. ***Leakage effects add distortion to the FRF and make it appear non-linear.*** These non-linear effects will cause curve fitting difficulties just as if the structure itself is non-linear.

Leakage Free Signals

The most effective way to make good FRF measurements is to use excitation signals that satisfy the assumptions listed above, so that the FFT will correctly transform them without leakage errors. Of course, both the excitation and response signals must comply with the above assumptions.

Transient Testing

In general, transient signals can be acquired so that both the excitation and response are completely contained within the sampling window. The analyzer can always be setup so that both the transient excitation and corresponding response signals are leakage free. In this case the analyzer would be setup to use no window. This is also refereed to as a Uniform, Rectangular or Boxcar window. If done properly, hammer (Impact) testing can yield FRF measurements devoid of any distortion or non-linear effects. [56], [59], [60].

Shaker Signals

The two most popular excitation signals used for shaker testing are random and sine. Again, to obtain leakage free FRF measurements, these signals (and their corresponding responses) must be either periodic or completely contained in the sampling window.

Two special types of random and sine signals that are commonly used are Burst Random and Burst Chirp. Burst Random is the same as a pure random (constantly changing) signal, but it is "turned off" prior to the end of the sampling window. This allows the structural response signal to decay to zero before the end of the sampling window. Therefore, both the excitation and response signals are completely contained in the sampling window.

Burst Chirp is a rapidly sweeping sine wave (chirp) signal that is setup in the analyzer to sweep over the frequency range of the FRF measurements. Burst Chirp is also "turned off" prior to end of the sampling window. This allows both the acquired force and response signals to be completely contained within the sampling window. Consequently, the resulting FRFs are leakage free.

Setting Up the Test Structure

When setting up the test structure, the first consideration usually faced is the boundary conditions imposed on the test structure. Boundary conditions are dictated by the purpose of the test. Will the results be used to verify a Finite Element Model, in which case "free-free" boundary conditions are used, or is the structure to be tested under "field conditions", so that a noise or vibration problem is recreated?

In general, boundary conditions will affect the dynamics of a structure, and hence its modes. Care should be taken not to introduce or excite any non-linearities in the structure by the imposed boundary conditions. Many structures have rattles (loose joints) that get excited when the excitation force is applied. These rattles can show up in the FRFs as noise or non-linear effects.

Shaker Testing

Shaker testing presents additional challenges when setting up the test structure. Not only must the shaker be attached to the structure, but the force that the shaker imparts to the structure must also be measured. Force is typically measured with a load cell.

A load cell only measures force in one direction, along its sensitive axis. To measure the force imparted to the structure, the load cell should be attached directly to the surface of the structure, and the shaker attached to the load cell in a manner that only imparts force through the sensitive axis of the load cell. This is usually accomplished by using a "stinger" or "quill", a slender rod that will support only axial loads.

If any unmeasured shear, bending, or torsional forces are imparted to the structure, the resulting FRF measurements are no longer the result of the measured axial force, but of multiple forces. Processing these measurements will once again lead to curve fitting difficulties and errors in the modal parameters.

CONCLUSIONS

The advent of the FFT algorithm in the late 1960's, and its implementation in digital test systems in the early 1970's has made the acquisition of vibration data and the calculation of FRF measurements fast and economical. During the past 20 years, a lot of research effort has been devoted to curve fitting algorithms for estimating modal parameters from experimental FRF data.

Our original paper (with the title of this paper) was presented at the first IMAC conference in 1982. Our paper documented the results of development work that was carried out in the late 1970's at the Hewlett Packard Co, and also in the early 1980's at Structural Measurement Systems.

Since then, we have implemented the RFP method in a variety of commercial modal analysis products, at Hewlett Packard, SMS, & Vibrant Technology. Others have also implemented it in their modal analysis packages [34], [35]. We have found the RFP to be a fast, relatively accurate, and reliable curve fitting method that is suitable for general use.

For this paper, we surveyed only the proceedings of the past 19 IMAC conferences, and found over 40 papers that referenced the RFP method. We found over 20 papers that extended the method, and added improvements to it.

Finally, since the quality of experimental modal parameter estimates depends so heavily on the quality of the FRF measurements used for curve fitting, we included some of our experience (gained over the past 20 years) making FRF measurements. Inaccurate FRF measurements can result for a multitude of reasons. We included brief discussions of the important sources of error in this paper. Some of these potential measurement problems are controllable and some are not.

In summary, the resulting set of experimentally derived FRFs that will be used for curve fitting must match the assumed analytical form of the FRF. Any deviation from this assumed form will lead to difficulties and errors in the curve fitting process, which translates into errors in the estimates of the modal parameters of the structure.

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