LOCATING OPTIMAL REFERENCES FOR MODAL TESTING

Brian Schwarz & Mark Richardson
Vibrant Technology, Inc.
Jamestown, California

Pete Avitabile
U. of Massachusetts
Lowell, Massachusetts

ABSTRACT

The Complex Mode Indicator Function (CMIF) was originally proposed as a method for improving modal parameter estimation. CMIF utilizes singular value decomposition (SVD) on a set of FRFs as a mechanism for extracting parameters.

The Multivariate Mode Indicator Function (MMIF) was originally proposed as a method for force appropriation to excite normal modes. MMIF utilizes an eigenvalue solution method on a set of FRFs to isolate modes.

Both CMIF & MMIF also have the ability to indicate the presence of closely coupled modes or repeated roots in a structure. Both methods can be used as tools for identifying a minimum set of reference DOFs for performing a modal test.

Using a small set of potential reference FRFs and their cross measurements, either CMIF or MMIF can be used to determine the minimum number of references required to adequately excite all the desired modes in a frequency band.

This technique has previously been demonstrated to provide reasonable results on several test structures. In this paper, the previous work is extended to provide a testing strategy for determining the number and locations of optimum references for modal testing. Examples are provided to demonstrate its use.

INTRODUCTION

One of the first questions that must be answered during the setting up of an Experimental Modal Analysis (EMA) or modal test is, "Where should the structure be excited, and where should its responses be measured?" This question breaks down into two questions,

- **Roving Excitation:** If the excitation is roving and the response fixed, how many fixed response DOFs are needed, and where should they be located.
- **Roving Response:** If the response is roving and the excitation fixed, how many fixed excitation DOFs are needed, and where should they be located.

When a single fixed (reference) excitation or response DOF is used, this is called a **single reference** test. If more than one reference excitation or response DOF is used, this is called a **multiple reference** test.

Single Reference Testing

Most modal testing is commonly done using either an impact hammer or shaker to excite the structure. In a single reference roving impact test, a single accelerometer is fixed to a DOF of the structure, and the structure is impacted at two or more different (roving) DOFs using the hammer.

In a single reference shaker test, a shaker is fixed to a DOF of the structure, and responses are measured at two or more different (roving) DOFs. The shaker is driven by a broadband signal, typically a random, transient, or swept sine signal.

In most modal testing, the structure is assumed to be "symmetric", obeying **Maxwell's reciprocity**. When this assumption is valid, a roving excitation & roving response test will yield equivalent modal parameter estimates.

Multiple Reference Test

In a multiple reference roving impact test, two or more accelerometers are fixed to different DOFs. In a multiple reference shaker test, two or more shakers are fixed to different DOFs.

A **reference**, then, is the **fixed** excitation or response DOF used to acquire data during a modal test. This paper will focus on determining the number and locations of the references required to identify all of the modes of interest of a structure.

What is Optimum?

The subject of this paper is locating optimum references. The term "optimum" implies that there is a single best solution to this problem. In most modal testing, there rarely is a single best solution. However, with respect to choosing references, we will see that "optimum" implies two things,

- Using as few references as necessary.
- Choosing reference DOFs that are not *at or near* nodal points of the mode shapes of interest.

NUMBER & LOCATION OF REFERENCES

In modal testing, a set of FRFs is typically acquired, from which experimental modal parameters are estimated by curve fitting the FRFs. An FRF is a 2-channel measurement. It is calculated between two signals, an excitation DOF and a response DOF signal.

No matter how the FRFs are acquired the following rule always applies,

> ***Mode Shape DOFs:*** *The roving DOFs of the FRFs in a set of FRF measurements define the DOFs of the mode shapes.*

For example, if a set of 100 FRFs is measured, a single reference set will yield mode shapes with 100 DOFs, a two reference set will yield mode shapes with 50 DOFs, a four reference set will yield mode shapes with 25 DOFs, etc.

Therefore, if time only permits the measurement of 100 FRFs, a single reference set of FRFs will yield mode shapes with better definition (more DOFS) than if multiple references are used. So, from the standpoint of defining mode shapes with the maximum number of DOFs, single reference testing should be preferred.

Avoiding Nodal Points

The first exception to this conclusion is one that every practitioner of modal testing understands from experience,

> *In a single reference test, the reference excitation or response DOF should be chosen where no mode shape is at a nodal point (a DOF where the mode shape magnitude is zero).*

Since the roving DOFs in a set of FRFs dictate the DOFs of the mode shapes, the reference DOF can be chosen *anywhere* on the structure without affecting the mode shapes. So the optimum single reference DOF is one where all of the modes of interest are not at nodal points.

USING ANALYTICAL MODES

If a Finite Element Model (FEM) of the test structure is available prior to testing, the model can be used to calculate a set of analytical mode shapes for the structure. These mode shapes can be used to locate one or more optimum references, where none of the mode shapes is at or near a nodal point.

Shape Product

A straightforward way to find an optimum reference is to calculate and display the shape product. The shape product is merely all of the mode shapes multiplied together, component by component. The result is a "shape" with the same DOFs as the original mode shapes, which can be displayed like a mode shape.

> *The shape product will have nodal points at all DOFs where the original mode shapes have nodal points.*

Figure 1 shows the mode shapes of the 92 & 142 Hz modes of a plate structure. Figure 2 shows their shape product. The dark line is the nodal line (locus of nodal points) of each shape.

It is clear from Figure 2 that the shape product contains the nodal lines of both mode shapes. It is also clear that either point **102** or **53** is an optimum reference for exciting the first two modes.

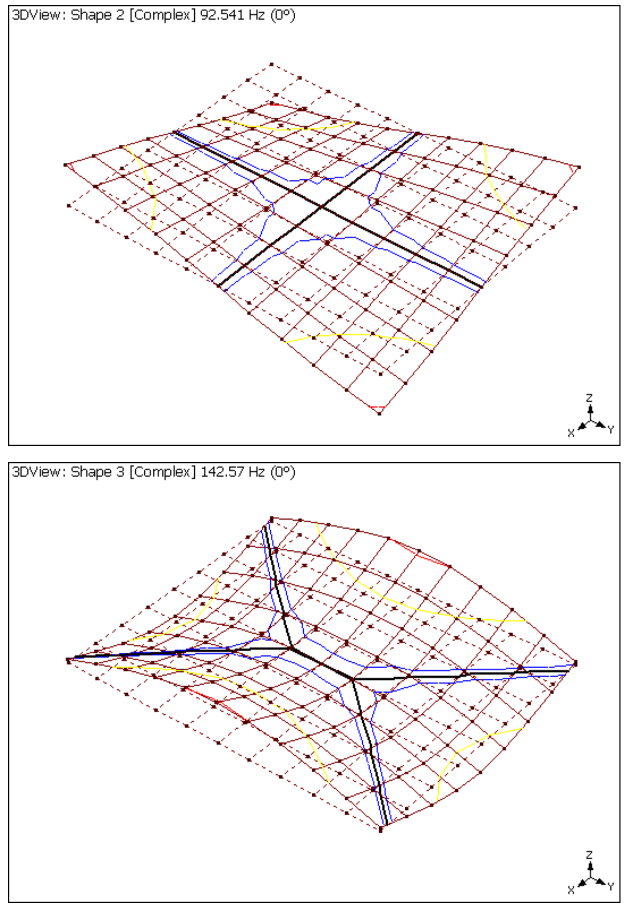

Figure 1. 93 & 142 Hz Mode Shapes With Nodal Lines.

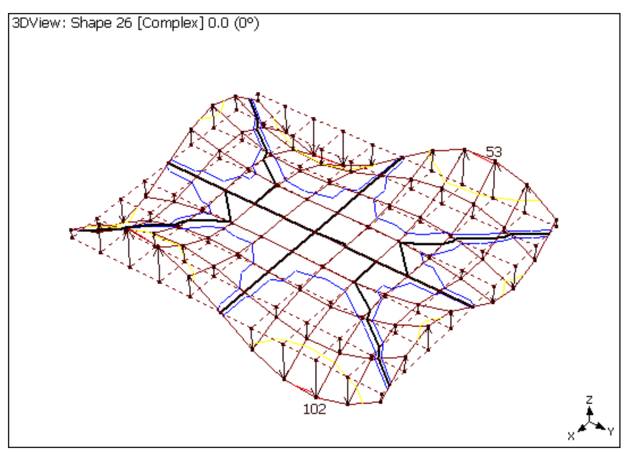

Figure 2 Shape Product of *93 & 142 Hz* Modes.

DRIVING POINT FRFS

Driving Point FRF. *A driving point FRF is any FRF where the excitation DOF is the same as the response DOF.*

In other words, a driving point FRF is calculated when the response is measured at the same point & direction where the excitation is applied.

Measuring FRFs

Driving point FRFs can be easily measured using an impact hammer, accelerometer, and 2-channel FFT analyzer. Each driving point FRF is obtained by attaching the accelerometer at a DOF and impacting the structure at (or near) the accelerometer location.

Synthesizing FRFs

Alternatively, a set of driving point FRFs can be calculated from a set of analytical mode shapes. A driving point residue for each mode (approximately the magnitude of the FRF at the modal frequency) is the *square* of the mode shape component at the driving point DOF [5].

A driving point FRF at or near a nodal point of a mode shape will have a small resonance peak or no peak at the frequency of the mode.

A large resonance peak is evidence that the mode is excited well at the driving point, and is not near a nodal point. On the other hand, a small resonance peak indicates that a mode is close to a nodal point. Clearly, any driving point where a mode is not represented with a large resonance peak relative to all others in the driving point FRF is not a good reference for a single reference test.

Displaying FRFs

A set of driving Point FRFs scaled relative to one another can be used to graphically determine one or more optimum references. Figure 3.A shows the log magnitudes of three driving point FRFs. Only the top FRF contains five resonance peaks, and therefore would be an optimum reference.

Figure 3.B shows a Cascade plot of the magnitudes of several driving point FRFs. Any of the driving point DOFs on this plot that has five large resonance peaks is an optimum reference for defining these five modes.

NEED FOR MULTIPLE REFERENCES

The previous methods may indicate that there is no single reference from which all modes can be excited, and therefore identified from a single reference set of FRFs.

The goal is to define experimental mode shapes with as many DOFs as possible, which means that as few reference DOFs as necessary should be used to acquire a set of FRFs.

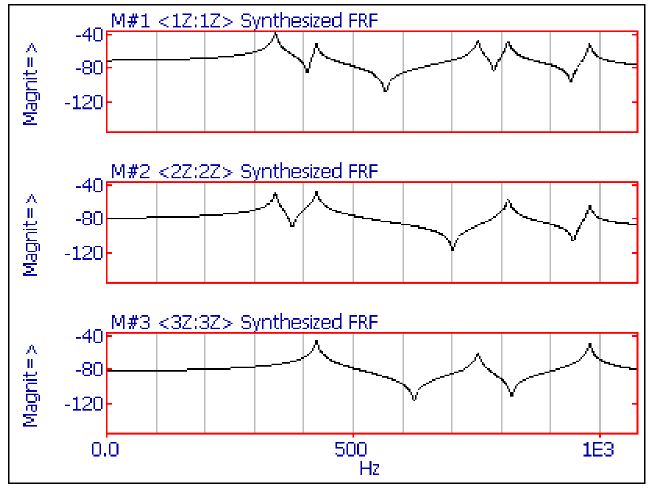

Figure 3.A Driving Point FRFs (relatively scaled).

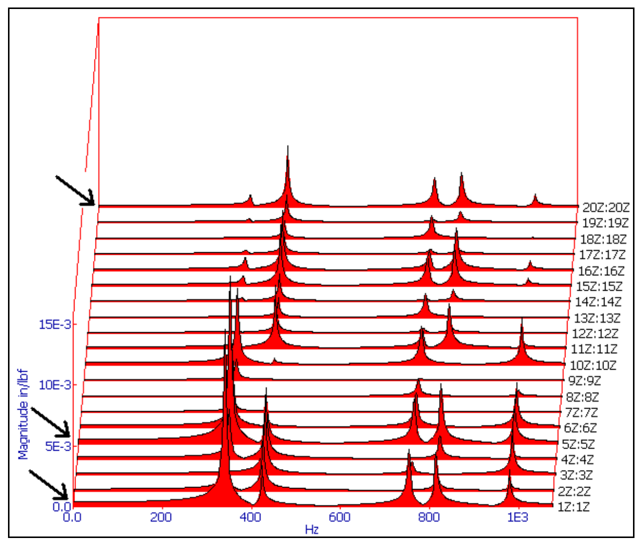

Figure 3.B Cascade of Driving Point FRFs.

Figure 4 shows the next two higher frequency modes (227 & 253 Hz) of the same plate structure that is shown in Figure 1.

Figure 5 shows the shape product of these two modes. The shape product indicates that points 100 & 55 are optimum references for identifying these two modes. However, it also shows that points 102 and 53 (optimum points for defining the 92 & 142 Hz modes) would be poor references for exciting the 227 & 253 Hz modes.

The driving point FRFs for DOFs 100Z and 102Z are shown in Figure 6. The upper FRF (driving point at 100Z) shows that the 142 Hz mode is poorly excited. The lower FRF (driving point at 102Z) shows that the 227 Hz mode is not excited from this reference.

The shape product and driving point FRFs both indicate that two references are required to optimally excite these four plate modes.

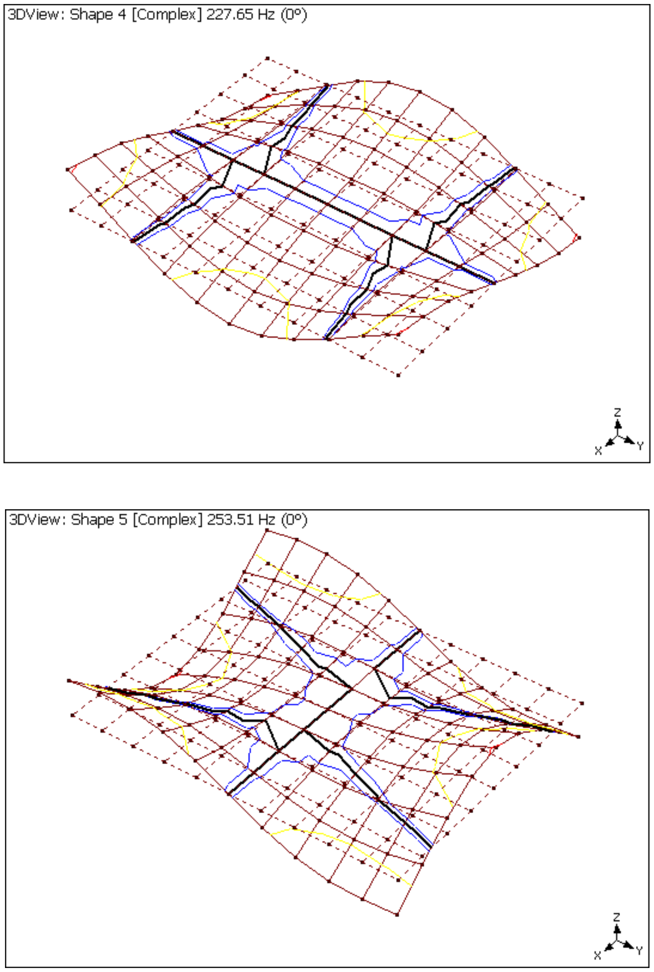

Figure 4. 227 & 253 Hz Mode Shapes.

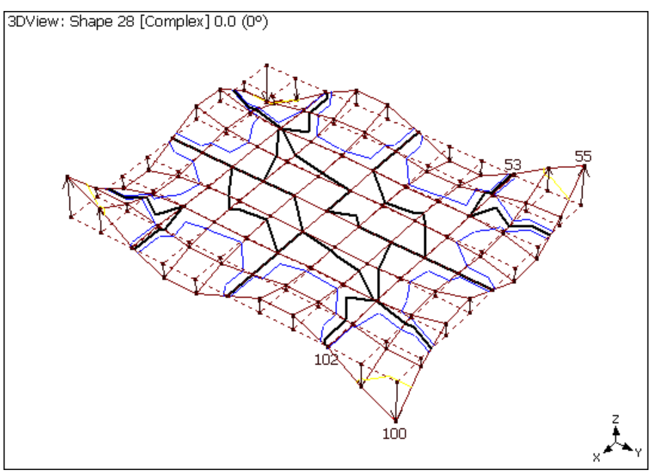

Figure 5. Shape Product of 227 & 253 Hz Modes.

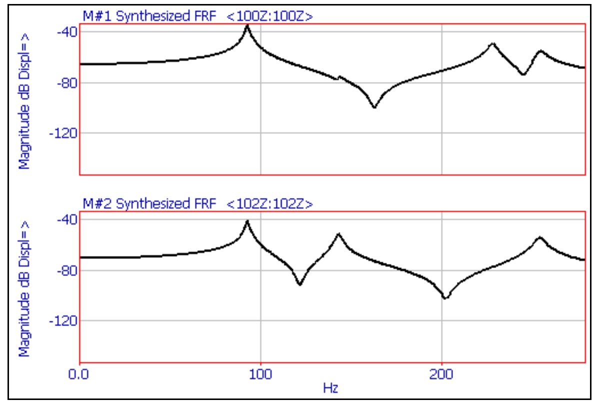

Figure 6. Driving Point FRFs for 100Z & 102Z.

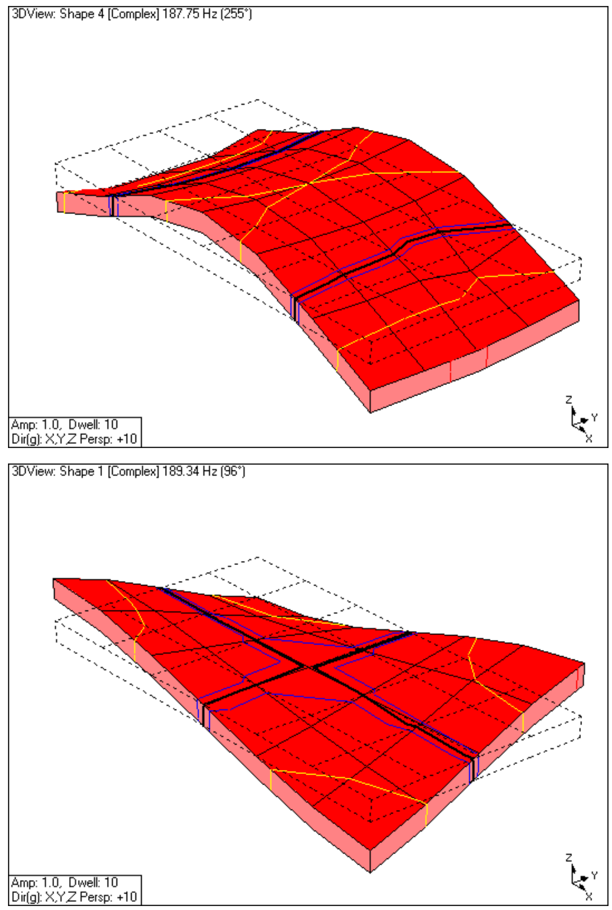

Figure 7. 187 & 189 Hz Mode Shapes.

CLOSELY COUPLED MODES AND REPEATED ROOTS

Many structures can have closely coupled modes or re-peated roots.

> ***Closely coupled modes*** *are two or more modes that are represented by a single resonance peak in a set of FRFs.*

> ***Repeated roots*** *are two or modes with the same modal frequency but different mode shapes.*

Repeated roots are also represented by a single resonance peak in a set of FRFs.

Figure 7 shows the first bending mode (at 187 Hz) and the first torsional mode (at 189 Hz) of a rectangular plate. These are closely coupled modes, as indicated in Figure 8 by the single resonance peak at 188 Hz in the overlaid FRFs.

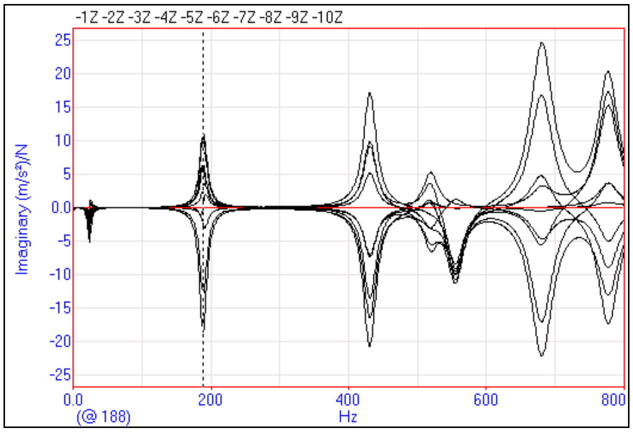

Figure 8. Overlaid Imaginary Part of FRFs Showing a Single Peak at 188 Hz.

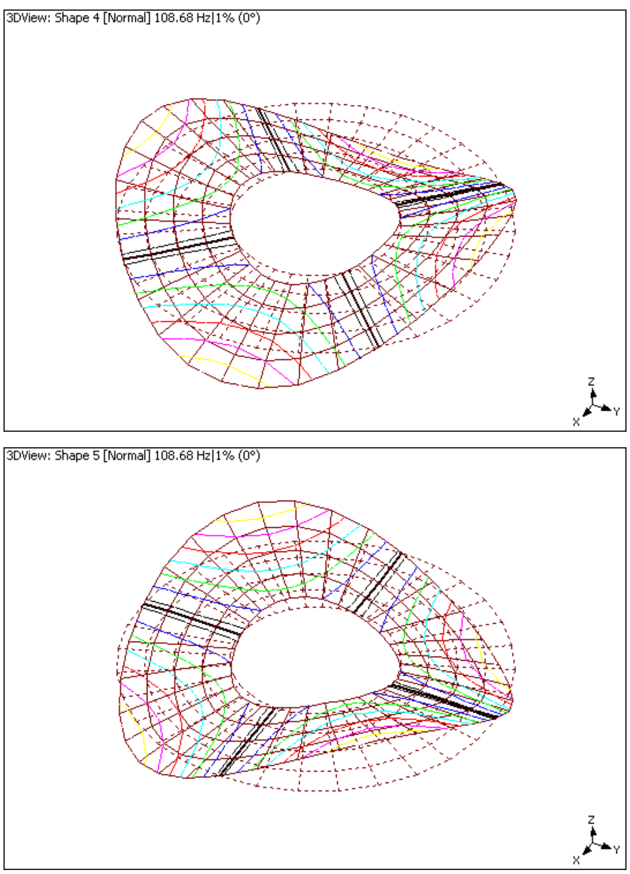

Figure 9. Two Repeated Roots at 108 Hz.

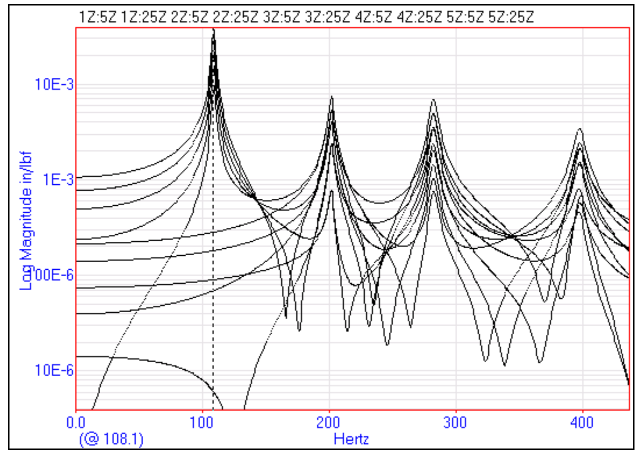

Figure 10. Overlaid Log Magnitudes of FRFs with Repeated Roots at 108, 282 & 397 Hz.

Figure 9 shows the mode shapes of two repeated root modes of a disk structure. Circular structures such as this often have repeated roots. Figure 10 shows overlaid FRFs that were synthesized using several of the disk modes. Clearly, there is only a single peak at **108** Hz. (In fact, this structure has pairs of repeated roots at **108, 282 & 397** Hz.)

For both of these cases, a multiple reference set of FRFs is required in order to identify the mode shapes of the closely coupled modes and repeated roots.

USING CMIF & MMIF

The Complex Mode Indicator Function (CMIF) was originally proposed as a method for improving modal parameter estimation [1], [4]. CMIF utilizes singular value decomposition (SVD) on a set of FRFs as a mechanism for extracting parameters.

The Multivariate Mode Indicator Function (MMIF) was originally proposed as a method for force appropriation to excite normal modes [2]. MMIF utilizes an eigenvalue solution method on a set of FRFs to isolate modes.

The Complex Mode Indicator Function (CMIF) and Multivariate Mode Indicator Function (MMIF) have previously been used for two purposes [1], [2], [7], [8],

- To indicate closely coupled modes and repeated roots.
- To provide modal participation factors for weighting multiple references during modal parameter estimation.

In this paper, both methods are used for a third purpose [3],

- To determine the optimum number of references necessary to identify all of the modes in a frequency band.

Even though these two methods use entirely different computational algorithms, they yield similar results. Each method calculates multiple resonance curves, where the number of curves equals the number of references of FRF data used in the calculation. Closely coupled modes or

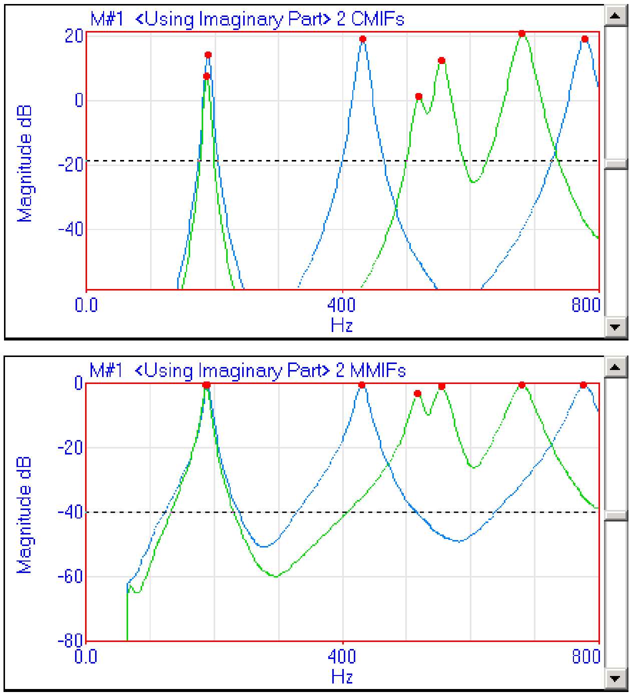

Figure 11. CMIF & MMIF Resonance Curves Showing Closely Coupled Modes at 188 Hz..

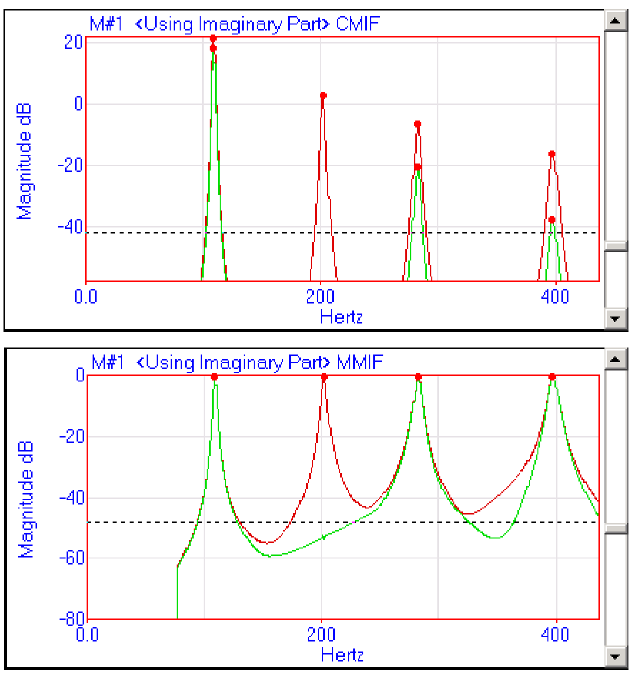

Figure 12. CMIF & MMIF Resonance Curves Showing Repeated Roots at 108, 282 & 397 Hz.

repeated roots are indicated by peaks on two or more of the resonance curves at or near the same frequency.

Figure 11 shows the CMIF & MMIF curves from a two-reference set of FRFs taken from the plate in Figure 7. Both methods indicate a total of 7 modes by the resonance peaks on their two curves, with two closely coupled modes at 188 Hz.

Figure 12 shows the CMIF & MMIF curves from a two-reference set of FRFs taken from the disk in Figure 9. These curves also indicate a total of 7 modes, with repeated roots (peaks on both curves) at 108, 282 & 397 Hz.

Both of these cases require at least a 2-reference set of FRFs to identify all of the modes in their respective frequency bands.

TESTING STRATEGY

Our proposed testing strategy starts with driving point FRF measurements and then uses CMIF or MMIF to check for closely coupled modes or repeated roots. Following are the steps for determining the optimum number of references required for a modal test,

- Measure **driving point FRFs** at some (ideally all) of the roving DOFs of a structure for which mode shapes are to be defined.
- Identify those **driving point FRFs** with the maximum number of strong (large magnitude) resonance peaks as potential references [5].
- If no single **driving point FRF** contains all of the peaks, find two or more **driving point FRFs** that contain all of the peaks. These FRFs then determine the number & location of the references.
- To check for **two** closely coupled modes or repeated roots, measure a **cross FRF** between two driving point DOFs selected from the previous steps. Calculate CMIFs and/or MMIFs, using the two driving point FRFs and the cross FRF. If closely coupled modes or repeated roots are indicated, use these DOFs for a 2-reference modal test.
- To check for **three** closely coupled modes or repeated roots, choose three driving point FRFs and all combinations of cross FRFs between these DOFs. (For three driving points, three additional cross FRFs must be measured.)

CONCLUSIONS

Several well-established methods for processing FRF measurements were brought together to define a testing strategy for determining the optimum number and locations of references for performing a modal test. The overall objective was to use as few references as possible, but also to insure that all of the modes of interest in a frequency band are adequately represented in the set of FRFs that are measured.

If a set of mode shapes is available prior to a modal test, it was shown that by displaying a shape products of the mode shapes, optimal references can be chosen that are not at or near nodal points of the mode shapes. It was also shown that even for a simple rectangular plate structure, a single optimum reference may be difficult to find.

Next, a simple procedure based on counting peaks in a set of driving point FRFs was discussed. One or perhaps several optimum references can be found by counting peaks in driving point FRFs.

Finally, is was shown that either the CMIF or the MMIF method can be used on a small set of FRFs to determine whether or not a multiple reference set of FRFs is required to identify closely coupled modes or repeated roots.

Experimental modal analysis relies strongly on acquired the best set of FRFs possible in order to estimate accurate modal parameters. If all of the modes of interest are strongly represented in a set of FRFs (with large magnitude resonance peaks at or near the driving points), then practically any modal parameter estimation (curve fitting) method can be applied to them to extract modal parameters.

REFERENCES

1. Shih, C.Y., Tsuei, Y.G., Allemang, R.J. & Brown, D.L. " Complex Mode Indicator Function and its Application to Spatial Domain Parameter Estimation" Proceedings of IMAC VII, January, 1989.
2. Crowley, R.W.J. & Vold, H. "The Multivariate Mode Indicator Function in Modal Analysis" Proceedings of the Third IMAC Conference, 1985.
3. Avitabile, P., Haselton, D. & Moore, J. " Modal Test Reference Selection Using an SVD Procedure" Proceedings of the 14th IMAC, 1996.
4. Luers, W., Deblauwe, F. & Lembregts, F. "Modal Parameter Estimation Based on Complex Mode Indicator Function " Proceedings of the 11th IMAC, 1993.
5. Kientzy, D., Richardson, M. & Blakley, K. "Using finite Element Data to Set Up Modal Tests" Sound and Vibration Magazine, June 1989.
6. Pickrel, C. R. "Numerical Assessment of Test Data Using SVD" Proceedings of the 15th IMAC, 1997.
7. Vibrant Technology, Inc. "Multiple Reference Curve Fitting To Find Repeated Roots" App. Note # 14, September, 2000.
8. Vibrant Technology, Inc. "Multiple Reference Curve Fitting To Find Closely Coupled Modes" App. Note # 15, April, 2001.