THE MULTIVARIATE MODE INDICATOR FUNCTION
IN MODAL ANALYSIS

Roger Williams
Structural Dynamics Research Corporation
York House, Stevenage Road
Hitchin, Hertfordshire SB4 9DY England

John Crowley
Structural Dynamics Research Corporation
2000 Eastman Drive
Milford, Ohio 45150

Havard Vold
Structural Dynamics Research Corporation
2000 Eastman Drive
Milford, Ohio 45150

ABSTRACT

In the sine dwell school of modal testing, a frequency domain function, most often called the Mode Indicator Function (MIF), has been employed for several years to detect the presence of real normal modes. This function is calculated from Frequency Response Functions from a single exciter location to highlight those frequencies where the global response phase lags the sinusoidal excitation by 90 degrees.

Recently, a multivariate mode indicator procedure was presented, that, using FRF's from several exciter locations, can detect repeated roots, as well as providing initial force patterns for the sine dwell tuning of the associated modes.

This paper presents a brief outline of the theory of this multivariate mode indicator and then shows how the information generated by this procedure may be used for the extraction of selected modal parameters through function enhancement and frequency domain SDOF techniques. The use of the multivariate mode indicator in the traditional MDOF procedures will also be discussed. Examples will be presented from several tests.

INTRODUCTION

An automatic method for finding optimum force patterns has been a key issue for the cost effective application of sine dwell modal testing. A mode is considered to be tuned cleanly when the acceleration response is 90 degrees out of phase with the excitation. In 1958, Asher [1] used this criterion to show that the determinant of the real part response matrix of a (square) frequency response matrix vanishes at a resonance frequency, and that the real force vector that spans the null space of this matrix is an optimal force pattern. This approach was used with modest success, probably because of the lack of adequate computer tools to measure and store accurate FRF measurements. Later, other researchers [2,3] extended Asher's method to rectangular FRF matrices and to modal tuning in a least squares sense.

In 1984, Vold [4] defined modal tuning as forcing to as low a value as possible, the ratio of the kinetic energies of in-phase response to total response. This minimum ratio, when plotted as a function of frequency, is called the Primary Mode Indicator Function, and its minima indicate the presence of modes that can be excited as real normal modes from the current set of exciter locations. The minimization problem that results from this approach turns out to be equivalent to finding the solutions of a generalized hermitian eigenvalue problem defined by an approximate mass matrix and the FRF matrix between the exciters and the response locations. In the theory section it is shown how this formulation produces higher order mode indicator functions and orthogonal force vectors, such that repeated modes can be detected and excited separately.

Since the mode indicator functions and associated force patterns are calculated from FRF matrices, this technology has also shown itself to be a valuable complement to the modern FRF based modal parameter extraction methods, such as Single and Multiple Degree Of Freedom (SDOF and MDOF) curve fitting. The mode indicator functions show the resonance frequencies, including repeated roots, and the force patterns may be used to enhance FRFs to make SDOF methods applicable to high modal density situations. In fact, this approach allows for an automated computer sine dwell simulation.

MATHEMATICAL BACKGROUND

The structural response $\mathbf{X(w)}$ is steady state for a purely real force vector $\mathbf{F(w)}$, and is given by

$$\mathbf{X(w)} = \mathbf{H(w)}\,\mathbf{F(w)} \qquad (1)$$

where $\mathbf{H(w)}$ is the frequency response function matrix. Dropping the frequency from the notation,

$$\mathbf{X} = \mathbf{HF} \qquad (2)$$

Expanding into real and imaginary components, equation (2) becomes,

$$X_r + iX_i = H_r F + iH_i F \qquad (3)$$

If a normal mode can be excited at a particular frequency, a force vector F must be found such that the real part X_r of the response vector is as small as possible compared to the total response. We define the norm of the real response by

$$\|X_r\|^2 = X_r{}^t MX_r \qquad (4)$$

where **M** is a mass matrix, either from a finite element model or a rough estimate of local mass distribution. This norm is proportional to a measure of kinetic energy. Likewise, the norm of the total response is given by

$$\|X_r + iX_i\|^2 = X_r{}^t MX_r + X_i{}^t MX_i \qquad (5)$$

The minimization problem is given by

$$\mathop{\|F\|=1}\limits^{min} \frac{\|X_r\|^2}{\|X_r + iX_i\|^2} = \mathbf{l} \qquad (6)$$

which after some algebra becomes

$$\mathop{\|F\|=1}\limits^{min} \frac{F^t AF}{F^t(A + B)} = \mathbf{l} \qquad (7)$$

where $A = H_r{}^t MH_r$, and $B = H_i{}^t MH_i$

Noting the similarity of equation (7) to a Rayleigh quotient, it can be shown that. the solution to equation (7) is found by finding the smallest eigenvalue $\mathbf{l}_{min}$ and corresponding eigenvector $\mathbf{F}_{min}$ of

$$AF = (A + B)F\mathbf{l} \qquad (8)$$

Plotting the smallest eigenvalue as a function of frequency gives a multivariate mode indicator function that clearly shows at which frequencies a normal mode exists that can be excited from the current set of exciter locations. Repeating the procedure for the second smallest eigenvalue reveals which frequencies, if any, are repeated modes. In those cases, the associated force eigenvector will be orthogonal to the first one, so that both modes can be tuned individually at the same frequency.

Consequently, all eigenvalues of equation (8) should be plotted as functions of frequency to show the multiplicity of normal modes.

EXPERIMENTAL APPROACH
In order to demonstrate the use of the multivariate MIF, one analytical model and several test structures were examined to provide a variety of conditions. Each physical structure was tested using a GenRad system running the DATM portion of the SDRC Modal-Plus software.

ANALYTICAL EXAMPLE
An analytical frame model with four modes of vibration was generated. FRFs were synthesized for 27 responses referenced to each of 2 driving points.

Figure 1 shows a driving point FRF derived from the analytical model. Figure 2 shows a plot of the primary MIF overlaid with the secondary MIF for the frame data. These MIFs are based on both reference locations and a set of response locations taken over the structure. All modes are indicated through the minima of the primary MIF, with the secondary MIF being inactive except for the two closely spaced modes. The slight drop in the secondary MIF is a result of the frequency resolution between the two modes. The resolution is sufficiently fine to separate the modes thereby producing only a slight drop. A coarser resolution would have forced a significantly lower value of the secondary MIF, indicating two modes within a spectral line.

CIRCULAR PLATE EXAMPLE
A circular plate was placed on a soft foam pad and accelerometers were placed at two points to measure response normal to the surface. Impact testing was chosen to acquire a modal survey. By using two stationary transducers, impacting around the structure, and assuming reciprocity, we obtain information equivalent to performing the test using two fixed exciter positions.

This geometrically symmetric structure was studied to examine the behavior of the multivariate MIF in the presence of repeated roots.

Figure 3 shows a driving point FRF obtained in the experimental test. Figure 4 shows a plot of the primary MIF overlaid with the secondary MIF for the circular plate test data. These MIFs are based on both reference locations and a set of functions taken over the structure.

The primary MIF has significant minima at four frequencies corresponding to modes of vibration of the plate. The secondary MIF has significant minima at three frequencies, corresponding to minima in the primary MIF. A drop in the primary and secondary MIF at these same

frequencies indicates the presence of repeated roots. A Polyreference analysis [5,6] verifies that these are indeed repeated roots.

For each mode indicated by the MIFs, the corresponding force pattern, as shown in Table 1, was employed to enhance the FRFs. A force pattern consists of an in-phase or out-of-phase force level for each reference (exciter location) used in the calculation of the MIF. An enhanced FRF for a given response is obtained by summing the product of the FRF, between that response and each reference, times the associated force level. Enhancing the FRFs enables Single Degree of Freedom (SDOF) techniques, in this case quadrature curve fitting, to be used.

As can be seen in Table 2, even for the repeated roots, the primary force patterns produces mode shapes orthogonal to the ones obtained by using the secondary force patterns.

BODY-IN-WHITE EXAMPLE
A body-in-white structure was mounted on four airbags which were attached to ground. The structure was excited using a pair of 50 pound shakers mounted vertically and symmetrically on the front of the structure. Load cells were mounted directly onto the structure, and thin metal rods about six inches long were used as stingers connecting the load cells to the shakers. To eliminate undesired motion of the excitation setup, a hot glue gun was used to fill in gaps between the shaker and any shims or blocking used for height adjustments.

A total of 35 sampling periods were taken using triggered random excitation. Baseband data was acquired up to 64 Hertz with 512 spectral lines of alias-free data.

Figure 5 shows a driving point FRF obtained in the experimental test. Figure 6 shows a plot of the primary MIF overlaid with the secondary MIF for the body-in-white test data. These MIFs are based on both reference locations and a set of functions taken over the structure. This structure exhibits a higher modal density than the preceeding examples as shown by the driving point function shown in figure 5. The minima of the primary MIF show how well the individual modes can be driven as real, normal modes from the current set of exciter locations. The secondary MIF in this example features slight drops only, suggesting no repeated roots.

AEROSPACE STRUCTURE EXAMPLE
An aerospace structure was excited using three 100 pound shakers mounted vertically. Load cells were mounted directly onto the structure, and nylon rods were used as stingers connecting the load cells to the shakers.

A total of 50 sampling periods were taken using pure random excitation and a 50$ overlap. Baseband data was acquired up to 16 Hertz with 512 spectral lines of alias-free data.

This example was chosen to illustrate an isolated root, a repeated root, and two very close roots all in a narrow frequency range. Figure 7 shows a plot of the primary MIF overlaid with the secondary MIF for this test data. A modest drop in the secondary MIF occurs at the closely spaced modes; whereas, at the repeated root the secondary MIF is almost as low as the primary MIF. The tertiary MIF exhibits no significant drops within this frequency range, indicating multiplicity of two at most.

CONCLUSIONS
The Multivariate Mode Indicator was originally developed as part of an activity to automate the tuning of real normal modes in sine dwell ground vibration testing. However, because of its ability to identify modes of vibration and provide forcing patterns to enhance those modes, MIF is applicable to other areas of modal testing. These applications include identifying roots and their multiplicity as well as allowing for the successful usage of SDOF methods in the presence of high modal density. The MIF technique complements the more advanced curve fitting algorithms in that it provides a phase and kinetic energy criterion for mode identification. In addition this MIF technique provides a field system alternative to advanced, compute intensive modal extraction algorithms by extending the applicability of SDOF methods.

REFERENCES
1) Asher, G. W., "A method of Normal Mode Excitation Utilizing Admittance Measurements", Proceedings, National Specialists' Meetings, IAS; Dynamics and Aeroelasticity (Institute of the Aerospace Aerospace Sciences, New York, November 1958)

2) Ibanez, P., "Force Appropriation by Extended Asher's Method", SAE Paper 760873, Aerospace Engineering and Manufacturing Meeting, San Diego, 1976.

3) Ibanez, P., Blakely, K.D.,"Automatic Force Appropriation A Review and Suggested Improvements", Proceedings of the Second International Modal Analysis Conference, Orlando, Florida, pp.903-907, February, 1984.

4) Hunt, D.L., Vold, H., Peterson, E.L., and Williams, R., "Optimal Selection of Excitation Methods for Enhanced Modal Testing", AIAA Dynamics Specialists Conference, Paper Number 84-1068 May, 1984.

5) Crowley, S. and Allemang, R., "Applications of the Poly Reference Technique in Experimental Modal

Analysis", Proceedings of the Second International Modal Analysis Conference, Orlando, Florida, pp.111-117, February, 1984.

6) Crowley, J., Rocklin, G. T., Hunt, D. L., and Vold, H., "The Practical Use of the Polyreference Modal Parameter Estimation Method", Proceedings of the Second International Modal Analysis Conference, Orlando, Florida, pp. 126-133, February, 1984

Freq. Hz	Reference		MIF
	1Y	6Y	Pattern
346.24	0.1044	0.9945	Primary
552.69	-0.8217	-0.5698	primary
737.63	-0.4825	0.8759	primary
1202.15	-0.7774	0.6289	primary
346.24	0.8980	0.4398	secondary
737.63	0.8699	0.4930	secondary
1202.15	0.2304	0.9731	secondary

Table 1. MIF Forcing Patterns   Circular Plate Test Structure.

| Freq.	346.24	552.69	737.63	1202.15	346.24	737.63	1202.15
346.24	1.00000	0.00007	0.00063	0.00004	0.00001	0.00133	0.00007
552.69		1.00000	0.00243	0.00753	0.00544	0.00674	0.00328
737.63			1.00000	0.00087	0.00087	0.00001	0.00081
1202.15				1.00000	0.00053	0.00027	0.00513
346.24					1.00000	0.00042	0.00019
737.63						1.00000	0.00053
1202.15							1.00000

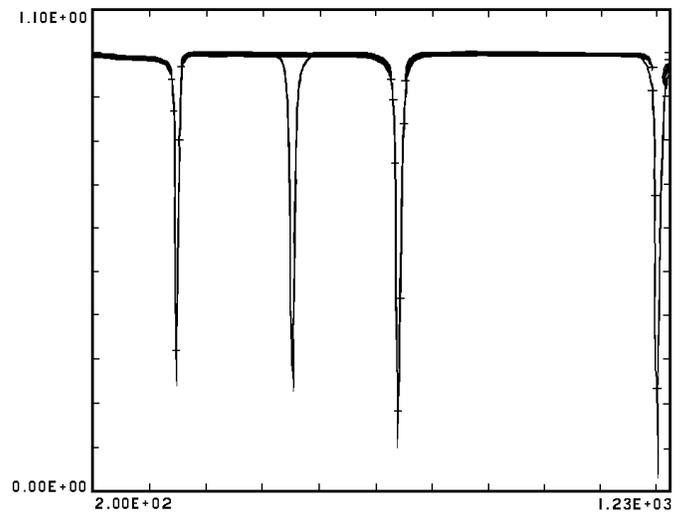**Table 2. Orthogonality Values   Circular Plate Test Structure.**
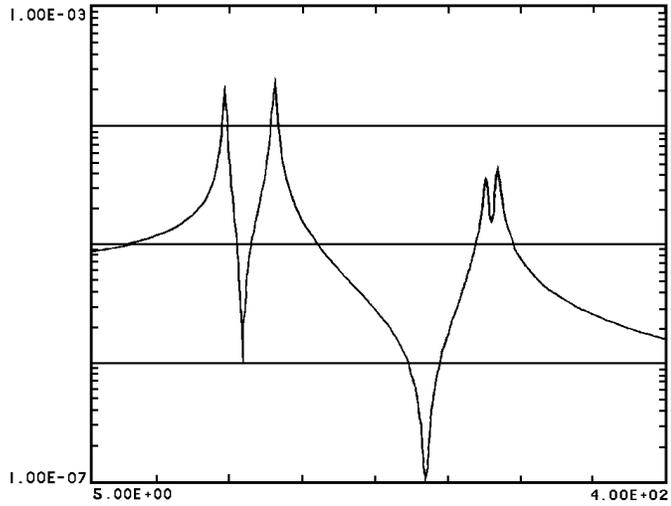
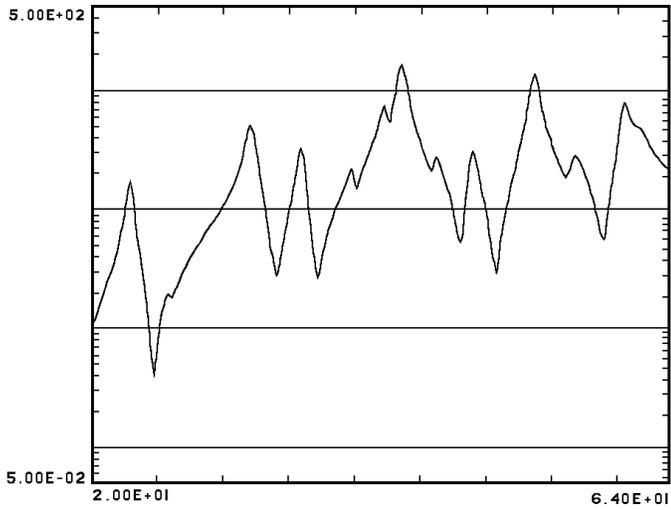**Figure 1. Analytical Frame Structure
Driving Point FRF.**
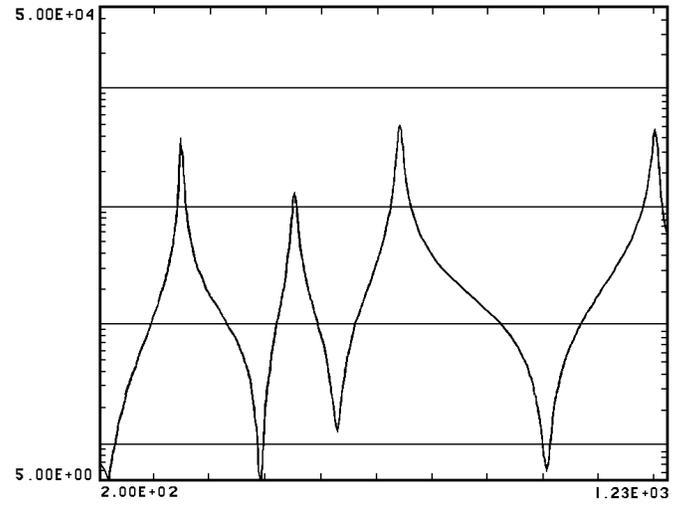
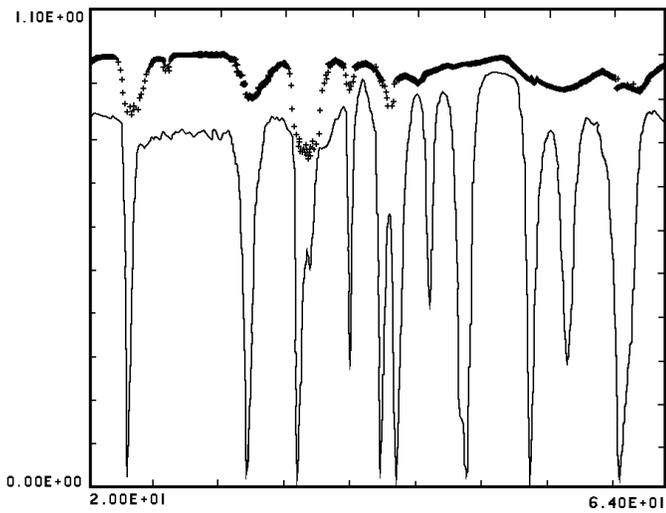**Figure 3. Circular Plate Test Structure
Driving Point FRF.**
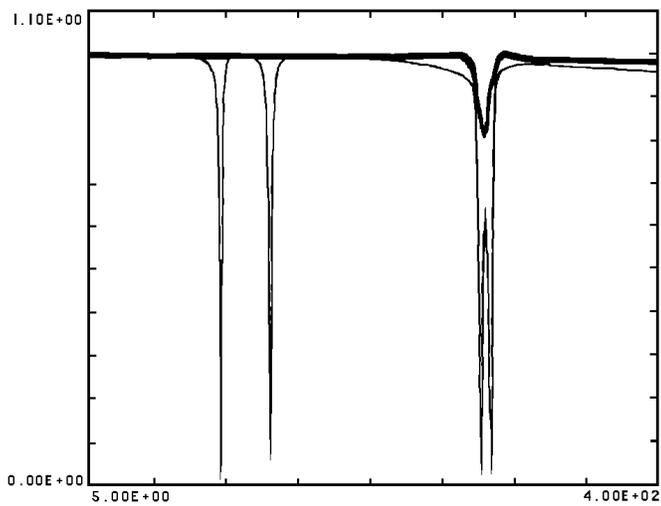
**Figure 2. Analytical Frame Structure
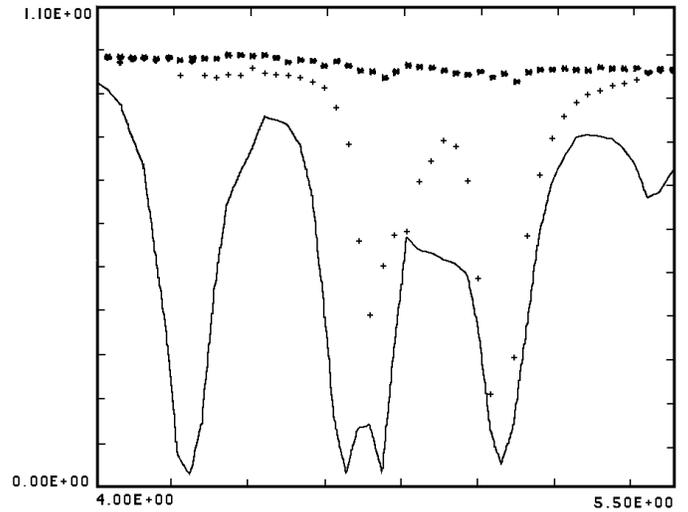Primary MIF, Solid Secondary MIF, Dots.**

**Figure 4. Circular Plate Test Structure
Primary MIF, Solid Secondary MIF, Dots.**

Figure 5. Body-In-White Test Structure Driving Point FRF.

Figure 7. Aerospace Test Structure Primary MIF, Solid Secondary MIF, Dots Tertiary MIF, Asterisks.

Figure 6. Body-In-White Test Structure Primary MIF, Solid Secondary MIF, Dots.