Complex Mode Indication Function and its Applications to Spatial Domain Parameter Estimation

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ABSTRACT

This paper introduces the concept of the Complex Mode Indication Function (CMIF) and its application in spatial domain parameter estimation. The concept of CMIF is developed by performing singular value decomposition (SVD) of the Frequency Response Function (FRF) matrix at each spectral line. The CMIF is defined as the eigenvalues, which are the square of the singular values, solved from the normal matrix formed from the FRF matrix, $[H(jw)]^H[H(jw)]$, at each spectral line. The CMIF appears to be a simple and efficient method for identifying the modes of the complex system. The CMIF identifies modes by showing the physical magnitude of each mode and the damped natural frequency for each root. Since multiple reference data is applied in CMIF, repeated roots can be detected. The CMIF also gives global modal parameters, such as damped natural frequencies, mode shapes and modal participation vectors. Since CMIF works in the spatial domain, uneven frequency spacing data such as data from Spatial Sine Testing can be used. A second stage procedure for accurate damped natural frequency and damping estimation as well as mode shape scaling is also discussed in this paper.

1. NOMENCLATURE

$\{f\}_r$ r^{th} mode shape.

l_r r^{th} system pole.

$m_k(jw)$ k^{th} eigenvalue of the normal matrix of FRF matrix.

jw frequency domain variable.

s_i i^{th} singular value.

$[\Phi]$ the mode shape matrix.

$[\sum(jw)]$ singular value matrix.

N degree-of-freedom of the system or the number of modes.

N_i number of excitation location.

N_k the number of repeated roots detected at jw_p.

N_o number of response point.

N_r the number of dominant modes.

Q_r scaling factor for r^{th} mode.

$[A_r]$ residue matrix for r^{th} mode.

$[M_\Phi]$ reduced mass matrix for response points.

$[M_L]$ reduced mass matrix for reference points.

$\hat{H}(jw)_p$ enhanced FRF for p^{th} mode.

$[H(jw)]$ frequency response function matrix.

$[\overline{H}(jw)]$ frequency response function matrix weighted by mass matrix.

$\{L\}_r$ r^{th} modal participation vector.

$[L]$ the modal participation factor matrix

$[U(jw)]$ left singular matrix.

$[V(jw)]$ right singular matrix.

2. INTRODUCTION

In the modal parameter estimation area, one of the greatest difficulties is to determine the number of degrees of freedom of the system in the frequency range of interest in order for the modal parameter estimation algorithm to be applied accurately. In time domain modal parameter estimation algorithms, such as the Ibrahim Time Domain (ITD) [1], the Polyreference Time Domain [2] and Eigensystem Realization Algorithm (ERA) [3] methods, an overdetermined system equation is found that will yield accurate results without leading to a numerically unstable characteristic equation. In frequency domain modal parameter estimation algorithms, such as Polyreference Frequency Domain [4] and Orthogonal Polynomial [5-8], the characteristic equation is not as numerically stable as in the time domain, and accurate results depend on the correct selection of the order of system equation.

A simple algorithm based on singular value decomposition (SVD) methods applied to multiple reference FRF measurements, identified as the Complex Mode Indication Function (CMIF), was first developed for traditional FRF data in order to identify the proper order of the system equation [8,9]. Unlike the Multivariate Mode Indication Function (MvMIF) [10], which indicates the existence of real normal modes, CMIF indicates the existence of real normal or complex modes and the relative magnitude of each mode. Furthermore, MvMIF yields a set of force patterns that can best excite the real normal mode while CMIF yields the corresponding mode shape and modal participation vector.

The CMIF is defined as the eigenvalues solved from the normal matrix formed from FRF matrix, at each spectral line. The normal matrix is obtained by premultiplying the FRF matrix by its Hermitian matrix as $\left[H(jw)\right]^H\left[H(jw)\right]$. The CMIF is the plot of these eigenvalues on a log magnitude scale as a function of frequency The peaks detected in the CMIF plot indicate the existence of modes, and the corresponding located frequencies of these peaks give the damped natural frequencies for each mode In the application of CMIF for traditional modal parameter estimation algorithms, the number of modes detected in CMIF determines the minimum number of degrees-of-freedom of the system equation for the algorithm.

The CMIF is not limited to be merely a mode indicator function. In the case that the smaller dimension of the FRF matrix is larger than or equal to the number of dominant modes at a specific frequency the singular value decomposition leads to modal vectors (mode shapes and modal participation vectors). The number of dominant modes at a specific frequency includes only the modes that have major contribution to the response at this frequency. The left singular vector is proportional to the mode shape, and the right singular vector relates to the modal participation vector. The mode shape obtained from CMIF is unscaled and the modal participation vector is relative as well (no absolute scaling is obtained).

The system pole (frequency and damping) and the sealed mode shape can be found in a second stage by using single degree-of-freedom algorithms on enhanced FRF measurements [11]. The enhanced FRF measurement data used here is the weighted summation of the FRF measurement data. The mode shape obtained from CMIF is used as the weighting function for the enhanced FRF measurement for each mode that is enhanced.

The CMIF is expected to be a powerful parameter estimation method for Spatial Sine Testing data. The concept of Spatial Sine Testing (SST) [12-16] has been introduced as the next generation of modal testing methods. This concept depends upon a description of the system frequency and/or impulse response function matrix in terms of the convolution of three fundamental characteristic variables; two spatial variables (input and response location), and one temporal variable (either time or frequency). Unlike frequency response methods which develop a database of frequency response functions, the SST method develops a database of forced modes of vibration (spatial information). Recently, low cost multiple input/multiple output data acquisition hardware has been implemented for large scale modal testing [17], which makes Spatial Sine Testing affordable. In Spatial Sine Testing multiple excitation involving up to 32 references is available. Assuming that more responses than inputs will be used, a maximum of 32 coupled modes (dominant modes including highly coupled modes or repeated modes) can be detected by the CMIF method.

3. THEORY

The FRF matrix describes the multiple input/multiple output relationship of the structure at each spectral line. In the modal analysis area, by assuming linear and time invariant systems, the FRF matrix of a N degree-of-freedom system can be expressed as

$$\left[H(jw)\right] = \sum_{r=1}^{2N} \frac{\left[A_r\right]}{jw - l_r} = \sum_{r=1}^{2N} \frac{Q_r \{f\}_r \{L\}_r^H}{\left(jw - l_r\right)} \qquad (1)$$

or in matrix form

$$\left[H(jw)\right] = \left[\Phi\right]\left[\frac{Q_r}{\left(jw - l_r\right)}\right]\left[L\right] \qquad (2)$$

where:

N_o is the number of response points.

N_i is the number of excitation points.

$[H(jw)]$ is the FRF matrix of size N_o by N_i

$[A_r]$ is the r^{th} residue matrix of size N_o by N_i

$\{f\}_r$ is the r^{th} mode shape of size N_o by 1

$\{L\}_r$ is the r^{th} modal participation factor of size N_i by 1

$[\Phi]$ is the mode shape matrix of size N_o by $2N$

$[L]$ is the modal participation factor matrix of size N_i by $2N$

Q_r is the scaling factor for r^{th} mode

l_r is the system pole value of r^{th} mode

In this expression, the response of the structure due to a unit excitation force at a particular frequency can be interpreted as a linear combination of the $2N$ residue matrices, weighted by the reciprocal of the distance between system pole l_r and the sampling frequency location jw in the Laplace domain. The residue matrix is the dyadic product of mode shape and modal participation factor, weighted by a scaling factor Q_r. Since the scaling of mode shapes and modal participation factors is arbitrary, the scaling factor obtained depends on the scaling of the mode shape and the modal participation factor. For the case where the mode shape and modal participation factor are scaled to be unitary vectors the scaling factor (Q_r) can be an indicator of the magnitude of the mode.

By taking the singular value decomposition of the FRF matrix at each spectral line, a similar expression as Eq.(2) is obtained.

$$[H(jw)] = [U(jw)]\left[\sum(jw)\right][V(jw)]^H \qquad (3)$$

where:

N_r is the number of dominant modes. The dominant modes are the modes that contribute to the response of the structure at this particular frequency jw.

$[U(jw)]$ is the left singular matrix of size N_o by N_r, which is a unitary matrix.

$\left[\sum(jw)\right]$ is the singular value matrix of size N_r by N_r, which is a diagonal matrix.

$[V(jw)]$ is the right singular matrix of size N_r by N_i, which is also a unitary matrix.

In order to compare Eq.(2) and Eq.(3), the mode shape and modal participation factor in Eq.(2) are scaled to be unitary vectors. Also, for simplification, the mass matrix in Eq.(3) is assumed to be an identity matrix; thus, the orthogonality of modal vectors is still satisfied

For the most usual case, the number of input points (reference points), N_i, is less than the number of response points, N_o. In Eq.(3), if the number of dominant modes is less than or equal to the smaller dimension of the FRF matrix, i.e. $N_r \le N_i$, the singular value decomposition leads to mode shapes (left singular vectors) and modal participation factors (right singular vectors). The singular value is then equivalent to the scaling factor divided by the distance between the sampling frequency point and the pole location. For the same mode, since the sealing factor is a constant, the closer the pole is to the sampling frequency point, the larger the singular value will be. Therefore the damped natural frequency is the frequency at which the maximum magnitude of the singular value occurs. In comparing different modes the larger the mode (larger residue value), the larger the singular value will be

The Complex Mode Indication Function is defined as the eigenvalues solved from the normal matrix, which is formed from the FRF matrix at each spectral line, $[H(jw)]^H[H(jw)]$. By this definition, the CMIF is equal to the square of the magnitude of the singular value. Therefore the peaks detected in the CMIF plot indicate the existence of modes, and the located frequencies give the corresponding damped natural frequencies

$$CMIF_k(jw) \equiv m_k(jw) = s_k^2(jw) \qquad k = 1,2,\cdots,N_r \quad (4)$$

and

$$[H(jw)]^H[H(jw)] = [V(jw)]\left[\sum{}^2(jw)\right][V(jw)]^H \quad (5)$$

where:

$CMIF_k(jw)$ is the k^{th} CMIF at frequency w.

$m_k(jw)$ is the k^{th} eigenvalue of the normal matrix of FRF matrix at frequency w.

$s_k(jw)$ is the k^{th} singular value of the FRF matrix at frequency w.

In practical calculations, the normal matrix formed from the FRF matrix, $[H(jw)]^H[H(jw)]$, is calculated at each spectral line; then, the eigenvalues of this matrix are obtained. The CMIF plot is the plot of these eigenvalues on a log magnitude as a function of frequency. An automatic peak detector based on preset criteria is used to identify the existence of modes. The eigenvector corresponding to the peak detected is equivalent to the modal participation factor. The unscaled mode shape (left singular vector) for the mode detected from the k^{th} eigenvalue curve, at frequency jw_p, can be solved as:

$$\{u(jw_p)\}_k = [H(jw_p)]\{v(jw_p)\}_k \, m(jw_p)_k^{-1} \qquad k = 1,2,\cdots,N_k \tag{6}$$

where

N_k is the number of repeated roots detected at jw_p.

jw_p is the frequency at peaks detected that is the approximate damped natural frequency of r^{th} mode

$\{u(jw_p)\}_k$ is the unscaled mode shape for k^{th} repeated root at jw_p.

$\{v(jw_p)\}_k$ is the equivalent mode participation factor for k^{th} repeated root at jw_p.

Once the unscaled mode shapes are obtained, the enhanced FRF for the p^{th} mode can be defined as:

$$\hat{H}(jw)_p \equiv \{u(jw_p)\}^H [H(jw)]\{v(jw_p)\} \tag{7}$$

Since the mode shapes from singular value decomposition are normalized to unitary vectors, by substituting Eq.(1) into Eq.(7), the enhanced FRF is actually the decoupled single mode response function:

$$\hat{H}(jw)_p \equiv \frac{Q_p}{(jw - l_p)} \tag{8}$$

Accurate damped natural frequencies and damping factors can be found by applying single degree-of-freedom modal parameter estimation methods to each enhanced FRF. The residue obtained in this single degree-of-freedom solution equals to the scaling factor Q_p; thus, the correct scaling of the mode shapes is obtained.

4. PRACTICAL CONSIDERATIONS

The peak in the CMIF indicates the location on the frequency axis that is nearest to the pole, and the frequency is the estimated damped natural frequency, to within the accuracy of the frequency resolution. The magnitude of the eigenvalue indicates the relative magnitude of the modes, residue over damping factor. It must be noted that not all peaks in CMIF indicate modes. Errors such as noise, leakage, nonlinearity and a cross eigenvalue effect can also make a peak. The cross eigenvalue effect is due to the way the CMIF is plotted. In a CMIF plot, the eigenvalue curves are plotted as a function of magnitude; the largest eigenvalue curve is plotted first; then, the other subsequent eigenvalue curves. Since the contributions from different modes vary along the frequency axis, at a specific frequency the contribution of two modes can be approximately equal. At this frequency these two eigenvalue curves cross each other. Because of the limited frequency resolution and the way that the CMIF is plotted the lower eigenvalue curve appears to peak, while the higher eigenvalue curve appears to dip. Therefore the peak in this case is not due to a system pole but is caused by an equal contribution from two modes. This characteristic is identifiable since the peak occurs in the lower eigenvalue curve at the same frequency as a dip in the higher eigenvalue curve. This effect is referred to as the cross eigenvalue effect. Another way of identifying the cross eigenvalue effect is to check the corresponding eigenvectors $[V(jw)]$ of the eigenvalue curves, i.e. the modal participation factors. The peaks that occur due to this cross eigenvalue effect can easily be identified by checking eigenvectors of adjacent spectral lines. If the eigenvectors of adjacent spectral lines do not represent the same shape as the eigenvector at the peak, this peak is not a system pole but is caused by the cross eigenvalue effect.

Since the dominant mode shapes that contribute to each peak do not change much around each peak, several adjacent spectral lines from the FRF matrix can be used simultaneously for a better estimation of mode shapes. By including several spectral lines of data in the singular value decomposition calculation, the effect of the leakage error can be minimized. If only the quadrature part of the FRF matrix is used in CMIF, the mode shape obtained turns out to be the same as in the Multi-MAC method [18].

In this paper, the mass matrix of the structure is assumed to be identity for simplification. But in the case of a real structure; the mass matrix is generally not an identity matrix. Therefore all formulations involving the FRF matrix, $[H(jw)]$, in this paper can be changed to weighted FRF matrices as follows:

$$[\overline{H}(jw)] \equiv [M_\Phi]^{\frac{1}{2}} [H(jw)][M_L]^{\frac{1}{2}} \tag{9}$$

where:

$\left[\overline{H}(jw)\right]$ is the weighted FRF matrix of size N_o by N_i.

$\left[M_\Phi\right]$ is the mass matrix of size N_o by N_o.

$\left[M_L\right]$ is the reduced mass matrix for reference points with size N_i by N_i.

The mass matrix will need to be obtained from other analytic methods such as Finite Element Analysis. The correct mass matrix is necessary for accurate mode shape estimation.

5. EXPERIMENTAL CASE

A Circular Plate data set of 6 references and 36 responses is used to demonstrate the use of CMIF. The geometry of the Circular Plate is shown in Fig. 1 with the reference point numbers circled. This data set was obtained by performing impact testing on the Circular Plate By assuming that Maxwell-Betti reciprocity exists, an FRF data set with multiple references is obtained. A typical FRF at one of the driving points is shown as in Fig. 2. The frequency resolution for this data set is 5 Hz. Since the geometry of the structure is symmetric, repeated roots are expected to exist in this structure. Therefore, multiple reference modal parameter estimation algorithms should be used for this data set.

Two cases which have been studied with the CMIF method give a representative example of the use of this method. In Case One, single spectral line data is chosen for each singular value decomposition. For this case, the CMIF plot detects nine modes as shown in Fig. 3, all in pairs of quasi-repeated roots except the third mode, which is a single mode (not repeated). For each mode detected the corresponding mode shapes can be calculated. The mode shapes obtained were then used as weighting factors in the calculation of each enhanced FRF. The enhanced FRF for the first mode is shown in Fig. 4. As a demonstration of being able to use spatial domain data with uneven frequency spacing, an orthogonal polynomial modal parameter estimation method is used to solve for the pole and residue from the enhanced FRF.

In Case Two, three adjacent spectral lines of data were chosen for each singular value decomposition. The CMIF plot is shown in Fig. 5. in which same nine modes were detected as in Case One. In comparing the CMIF plots from the two cases, as more spectral line data is included in the singular value decomposition procedure the eigenvalue curves are smoother, since more averaging is performed. In Fig. 5, mode 4 and mode 5 were detected in the same spectral line rather than in different spectral lines as in Fig. 4. The same procedure of FRF enhancement and of single degree-of-freedom modal parameter estimation was used for Case Two.

In addition to the CMIF method, the Polyreference Time Domain Method is also applied on the same data set for comparison purposes. In Table 1, the estimated mode shapes from both CMIF cases were compared to the corresponding mode shapes from Polyreference Time Domain Method by Modal Assurance Criterion (MAC) [11] value. Note that in the case of repeated roots, due to symmetric geometry of the structure, the corresponding mode shapes can be chosen by the algorithm arbitrarily, as long as the corresponding vectors span the same subspace. Therefore, the mode shapes of repeated roots cannot be compared by MAC value directly. Since the repeated root mode shapes found from the CMIF are chosen orthogonal to each other, one simple way of comparing mode shapes associated with repeated roots from different algorithms is to calculate the partial coherence of the mode shape to a plane which is spanned by the mode shapes corresponding to repeated roots. The partial coherence of the i^{th} mode shape to the plane spanned by two modes, mode 1 and 2, is defined as $1.0 - MAC_{i1} - MAC_{i2}$. In Table 1, the mode shapes of repeated roots that are compared by partial coherence were indicated by a "*" sign in front. In comparing these results, the mode shapes from both CMIF cases were in agreement with mode shapes from Polyreference Time Domain Method. Case Two has somewhat better estimation of mode shapes, since more spectral lines of data were averaged in each singular value decomposition.

The frequency and damping estimations of these three cases are listed in Table 2. The pole estimations are reasonable compared to the Polyreference Time Domain Method, except for mode 4 in both CMIF cases. This mode was not well excited as shown by the smaller eigenvalue in the CMIF plot for both cases. Note that a mass matrix was not available for the CMIF cases. For this reason and since the frequency resolution of the data was 5 Hz., the accuracy of the CMIF cases compared to the Polyreference Time Domain Method was quite good.

6. CONCLUSION

In conclusion, a singular value decomposition technique has been applied to the FRF matrix at each spectral line. The comparison between singular values/vectors and modal parameters is made in order to interpret the physical significance. The CMIF is defined as the eigenvalues solved from the normal matrix of the FRF matrix. The CMIF plot is the plot of these eigenvalues in log scale. The peaks detected in CMIF plot indicate the existence of modes, the located frequency is the approximate damped natural frequency, and the corresponding left and right singular vectors are related to mode shape and modal participation vector respectively. In a second stage procedure, the damped natural frequency, damping and residue can be obtained by applying any single degree-of-freedom modal parameter estimation algorithms to the enhanced FRF. With these modal parameters and additional mass matrix information, the mode shape can be scaled.

The CMIF shows some attractive features in the modal parameter estimation area. Some of the advantages are:

- CMIF identities the number of modes of the system and the existence of repeated roots before a traditional modal parameter estimation algorithm is applied.
- The eigenvalues can be used as a weighting function or simply as an index for selecting multiple frequency bandwidths for a frequency domain parameter estimation algorithm.
- CMIF shows the physical magnitude of each mode as excited from the references used in the FRF data.
- CMIF is a straightforward procedure. The requirement of user judgment and experience is minimized.
- By using the singular value decomposition technique the CMIF is robust for noise contaminated data.
- Data over several spectral lines can be included in the singular value decomposition in order to reduce the effects of errors, such as leakage in the data.
- CMIF is good for uneven frequency spacing data, such as Spatial Sine Testing data.

There are also some limitations in the application of CMIF:

- Multiple reference FRF information is needed for the CMIF calculation. The number of dominant modes at each spectral line is assumed to be less than the number of references. Therefore the modal density as a function of frequency resolution is limited by the number of reference points.
- The accuracy of the estimated CMIF modal parameters is limited by the frequency resolution.
- The knowledge of a reduced mass matrix is needed for a more accurate CMIF calculation.
- An additional second stage procedure is needed for scaled mode shapes and more accurate pole estimation.

7. REFERENCES

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Mode Shape Estimation (MAC)		
Mode	Polyreference to CMIF-1	Polyreference to CMIF-3
1	* .9996	* .9998
2	* .9999	* .9999
3	.9993	.9995
4	.9987	* .9994
5	.9998	* 1.000
6	* .9968	* .9969
7	* .9996	* .9997
8	* .9991	* .9991
9	* .9979	* .9979

Notes:
* MAC value estimated between one mode shape and a plane that is spanned by two mode shapes (repeated mode).

TABLE 1. Comparison of estimated mode shapes between Polyreference and CMIF by MAC.

Frequency / Damping Estimation						
Mode	Polyreference		CMIF - 1		CMIF - 3	
	Frequency * (Hz)	Damping (%)	Frequency * (Hz)	Damping (%)	Frequency * (Hz)	Damping (%)
1	362.467	0.8717	361.642	0.5877	363.209	0.9448
2	363.768	0.9195	363.398	0.9457	365.305	0.9945
3	557.057	0.5541	557.129	0.5165	557.129	0.5164
4	761.228	0.6935	756.981	1.4138	738.941	0.7075
5	764.163	0.3308	764.185	0.3615	764.185	0.3614
6	1223.05	0.3457	1222.85	0.2823	1222.47	0.2716
7	1224.09	0.3305	1223.00	0.3338	1223.01	0.3337
8	1328.33	0.4813	1327.98	0.4445	1328.05	0.4280
9	1328.94	0.3989	1328.35	0.5159	1328.45	0.5225

Notes:
* Data set with 5 Hz frequency resolution
CMIF - 1 CMIF use of single spectral line FRR data
CMIF - 3 CMIF use of three spectral lines FRF data for each singular value decomposition

TABLE 2. Comparison of estimated frequency and damping by Polyreference Time Domain and CMIF method

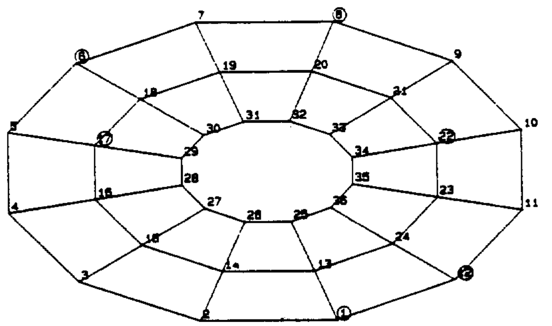

Figure 1. The "Circular Plate" Structure with Reference Points.

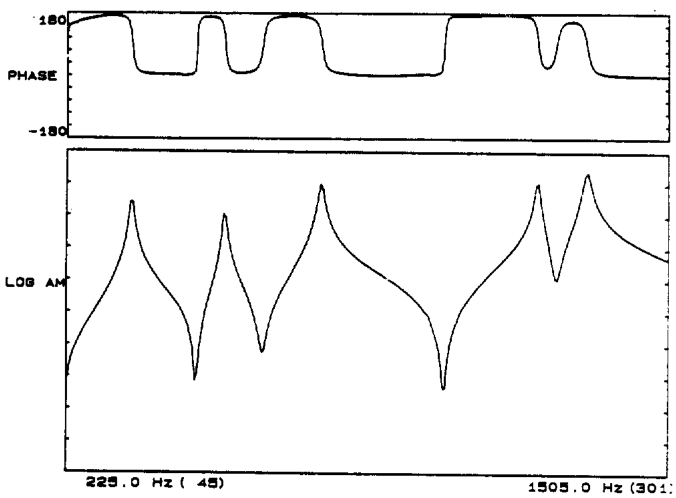

Figure 2. The Driving Point Frequency Response Function.

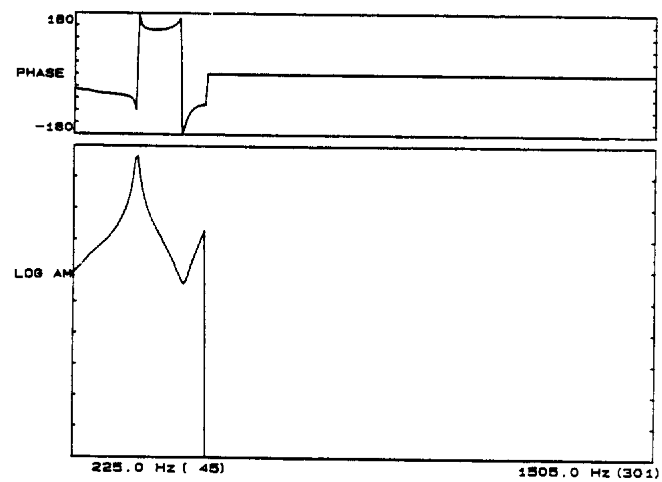

Figure 4. The enhanced FRF for the first mode.

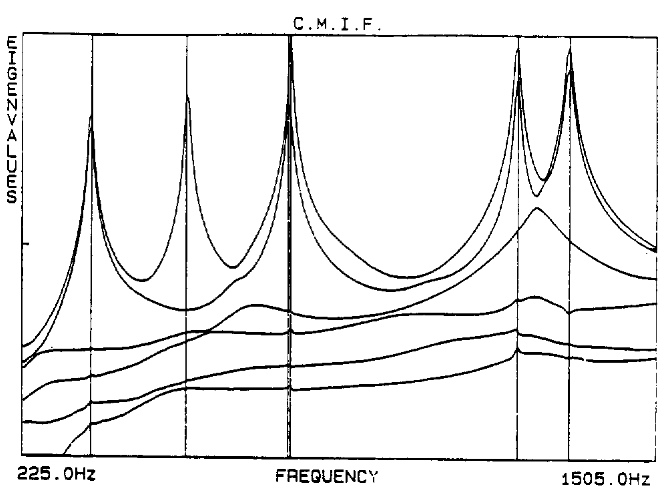

Figure 3. The CMIF Plot use single spectral line FRF data.

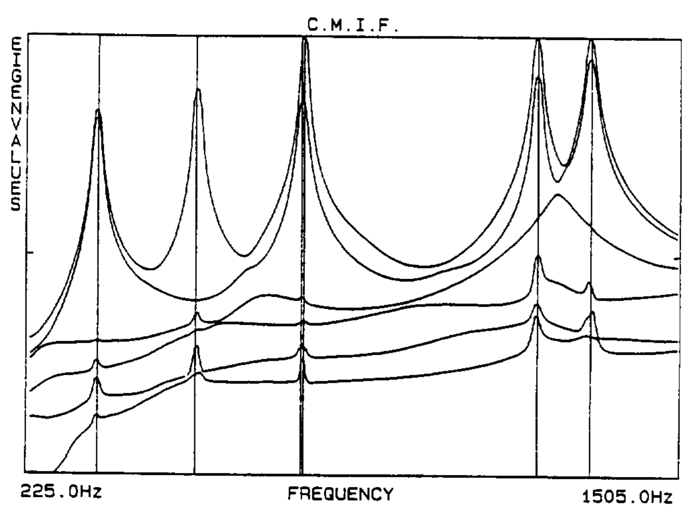

Figure 5. The CMIF Plot use three spectral lines of FRF data.