Structural Dynamics Measurements

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Introduction

In this paper, the term "structural dynamics measurements" will more specifically mean the ***measurement of the vibration of mechanical structures and machinery***. Because this topic is so broad in scope, modal analysis and signal processing are also discussed here, but other papers at this conference are specifically devoted to those topics and cover them in more detail.

Why Vibration Measurements?

Why are vibration measurements important? Because vibration contributes to a variety of undesirable behavior in machinery and structures. A machine or structure,

- may be ***uncomfortable*** to ride in,
- is ***too difficult to control***,
- makes ***too much noise,***
- doesn't ***maintain tolerances,***
- ***wears out*** too fast,
- ***fatigues*** prematurely,
- or ***breaks*** unexpectedly.

Types of Vibration

All structural vibration can be characterized as a combination of ***forced*** and ***resonant*** vibration. No vibration can occur at all unless forces are applied to the structure. However, resonant vibration can still occur after the forces have been removed. Resonant vibration is also conveniently characterized in terms of the ***modes of vibration*** of a structure.

Resonant Vibration

A structure's modal parameters (resonant ***frequency, damping,*** and ***mode shape***) can be estimated from certain kinds of structural dynamics measurements. If excited, modes (or resonances), can act like "***mechanical amplifiers***". Modes can cause excessive vibration responses that are ***orders of magnitude*** greater than responses due to static loading.

Key Issues in Structural Dynamics Testing

Since dynamic behavior can be unpredictable due to the excitation of structural resonances, the most important question to be answered from structural dynamics testing is,

- "Is a ***structural mode*** being excited?"

In addition, several other questions need to be answered,

- "What are the ***excitation forces,*** and where are they coming from?"
- "Is the system ***non-stationary***?"
- "Is the system ***non-linear***?"

Modes are only defined for ***linear, stationary*** mechanical systems. Most real structures can exhibit non-stationary (***non-steady state***) and non-linear dynamic behavior. When testing for the modes of a structure, these issues and others must be taken into account.

Spectral Analysis

Probably the most convenient way to analyze a vibration signal is to obtain its frequency content, or frequency spectrum. There are at least two good reasons for this,

1. Excitation forces (especially in rotating equipment), often provide sinusoidal excitation at specific frequencies. These forces are manifested as peaks in a frequency spectrum.

2. Resonances are also manifested as peaks in a frequency spectrum.

Prior to the late 1960's, all structural dynamics testing was done with analog instrumentation. Sine wave generators were used to artificially excite structures, one frequency at a time. Oscilloscopes were used to look at the signals. Analog filters were used to limit the frequency content (band limit) the signals. Special analog filters that changed with the frequency of excitation (swept filters), were used to obtain the structural response, one frequency at a time. This response to each sinusoidal excitation frequency is a ***frequency spectrum***.

In the 1960's, commercial spectrum analyzers were marketed that utilized swept analog filters, and constructed the frequency spectrum of structural vibrations, one frequency at a time.

The FFT Analyzer

The Fourier transform is a mathematical procedure that was invented by a Frenchman named Jean-Baptiste-Joseph Fourier in the early 1800's. The Fourier transform yields the frequency spectrum of a time domain function. It is defined for continuous (or analog) functions.

The discovery of the Fast Fourier Transform (FFT) algorithm in the late 1960's opened up a whole new area of signal processing using a digital computer [1]. The FFT computes a discretized (sampled) version of the frequency spectrum of a sampled time signal. This discretized, finite length spectrum is called a Discrete Fourier Transform (**DFT**). Following its discovery, the FFT was implemented in a new kind of spectrum analyzer called an FFT, or Fourier analyzer.

Present day FFT analyzers can compute a DFT in milliseconds, whereas it used to take hours using standard computational procedures. From a DFT, FFT analyzers can calculate a variety of other frequency domain functions, including Auto Power Spectra (**APS**s), Cross Power Spectra (**XPS**s), Frequency Response Functions (**FRF**s), Coherences, etc.

Rules of Digital Measurement

There are three key equations that govern the use of the DFT. The first one describes the sampled signal in the time domain, the second describes the sampled spectrum in the frequency domain, and the third is Shannon's Sampling Theorem, also called the Nyquist sampling rate.

Time Waveform: The DFT assumes that the sampled time waveform contains **N** uniformly spaced waveform samples, with an increment of (**Δt**) seconds between samples. (The most common FFT algorithms restrict N to being a power of 2, although this is not necessary.) The total time period of sampling (also called the ***sampling window***), starts at (**t = 0**) and ends at (**t = T**). Therefore,

$$T = (\Delta t)\, N \quad \text{(seconds)}$$

Frequency Waveform: The DFT assumes that the digital frequency spectrum contains **N/2** uniformly spaced samples of complex valued data, with frequency resolution (**Δf**) between samples. The frequency spectrum is defined for the ***frequency range*** (**f = 0**) to (**f = Fmax**). Therefore,

$$F_{max} = (\Delta f)\,(N/2) \quad \text{(Hertz)}$$

Nyquist Sampling: Shannon's Sampling Theorem says that a frequency spectrum can only contain unique frequencies in a range from (**f=0**) up a maximum frequency (**f = Fmax**) equal to ***one half the sampling rate*** of the time domain signal. Therefore,

$$F_{max} = (1/2)\,(1/\Delta t) \quad \text{(Hertz)}$$

Fundamental Rule: To Improve Frequency Resolution, You Have to Wait

The three equations above can be used to derive the most fundamental rule of digital spectrum based testing,

$$\Delta f = (1/T)$$

This equation says that the frequency resolution obtainable in a digital spectrum ***depends on the time domain sampling window length*** (**T**), not the sampling rate. Stated differently, to get better frequency resolution, you have to sample over a longer time period.

Zoom Measurements

A popular digital signal processing technique that is implemented in most FFT analyzers is the Zoom transform, or Zoom measurement. A Zoom transform is essentially a digital filtering operation that takes place after the time waveform has been sampled. It involves re-sampling, frequency shifting, and low pass filtering of the sampled data to yield a DFT with increased frequency resolution, but over a smaller frequency band.

The Zoom transform is very useful for obtaining better frequency resolution without having to perform an FFT on a very large number of samples. From a practical standpoint, the Zoom transform is much faster than using a base band FFT (starting at zero frequency) with more samples to obtain more frequency resolution.

As an example, in order to obtain 1 milli-Hz of resolution in the vicinity of 100 Hz, a base band FFT would have to transform at least 262,144 samples. This would yield a base band spectrum between 0 and 132 Hz.

$$132\ \text{Hz} = (0.001\ \text{Hz})\,(262{,}144 / 2)$$

Even though the Zoom transform starts with the same 262,144 time samples, the Zoom band can be centered around 100 Hz, and, assuming that a 1 Hz bandwidth is sufficient, the FFT only needs to transform 2048 samples,

$$1\ \text{Hz} = (0.001\ \text{Hz})\,(2048 / 2)$$

Digital Measurement Difficulties

The rules above are basically all that is required to make digital measurements. However, there are two remaining difficulties associated with the use of the FFT. They are called aliasing and leakage.

Aliasing

Aliasing of a signal occurs when it is ***sampled at less than twice the highest frequency*** in the spectrum of the signal. When aliasing occurs, the parts of the signal at frequencies above the sampling frequency add to the part at lower frequencies, thus giving an incorrect spectrum.

All modern FFT analyzers guarantee that aliasing will not occur by passing the analog signals through ***anti-aliasing filters*** before they are sampled. An anti-aliasing filter band limits (low pass filters) the signal so that it contains no frequencies higher than the sampling frequency. Since all filters have a roll off frequency band, the cutoff frequency of the anti-aliasing filters is typically set to ***40% of the sampling frequency***. Therefore, ***80% of a DFT frequency band*** is considered to be alias-free.

Leakage

The FFT assumes that the signal to be transforming is ***periodic in the transform window***. (The transform window ***is*** the samples used by the FFT). To be periodic in the transform window, the waveform must have no discontinuities at its beginning or end, if it were repeated outside the window.

Signals that are always periodic in the transform window are,

1. Signals that are completely contained within the transform window.

2. Cyclic signals that complete an integer number of cycles within the transform window.

However, many other types of signals (such as random signals), may not be periodic in the transform window. If a time signal is ***not periodic*** in the transform window, when it is transformed to the frequency domain, a ***smearing*** of its spectrum will occur. This is called leakage. ***Leakage distorts the spectrum and makes it inaccurate***.

Minimizing the Effects of Leakage

If a signal is non-periodic in its sampling window, it will have leakage in its spectrum. In this case, leakage can never be eliminated but it can be minimized. To minimize the effects of leakage, specially shaped windows are applied to the time waveforms ***after*** they are sampled, but ***before*** they are transformed using the FFT.

- **Hanning Window:** The Hanning window is effective for minimizing the effects of leakage in the spectra of ***broad band signals***, such as random signals.
- **Flat Top Window:** The Flat Top (Potter P301) window is effective for minimizing the effects of leakage in the spectra of ***narrow band signals***, such as sinusoidal signals.
- **Exponential Window:** This window is effective for minimizing the effects of leakage in impulse responses that don't damp out within the sampling window.

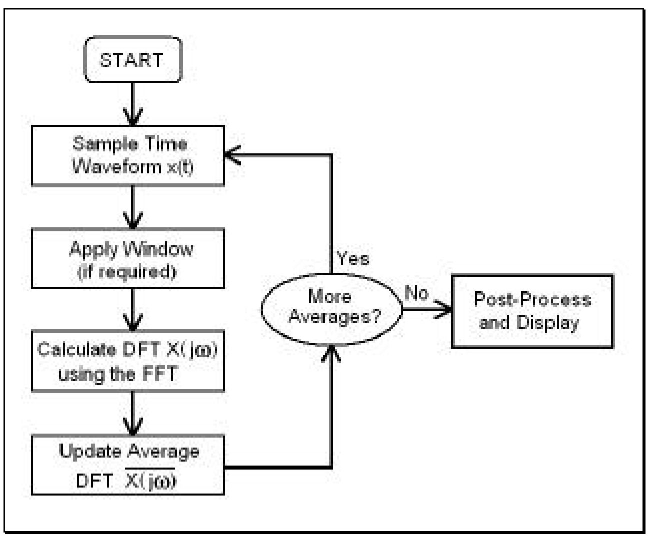

Figure 1. Spectrum Averaging Loop

Spectrum Averaging

Spectrum averaging in an option in most modern FFT analyzers. It is done with a spectrum averaging loop, as shown in Figure 1.

Spectrum averaging is used to remove the effects of,

1. ***extraneous random noise***.
2. ***randomly excited non-linearities***.

In a spectrum averaging loop, multiple spectral estimates of the same signal are averaged together to yield a final estimate of the spectrum. Different types of averaging can be used, but the most common type (called ***stable averaging***), involves summing all of the estimates together and dividing by the number of estimates.

The FFT is a linear, one-to-one and onto transformation. That means that it ***uniquely transforms*** the vibration signal from a linear dynamic system into its correct digital spectrum, and vise versa. If a signal contains any ***additive Gaussian random noise or randomly excited non-linear behavior***, these portions of the signal are transformed into spectral components that ***appear randomly in the spectrum***.

Removing Random Noise & Non-Linearities

By summing together (averaging) multiple spectral estimates of the same signal, the linear spectral components will add up (re-enforce one another), while the random noise and non-linear components will sum toward zero, thus removing them from the resultant average spectrum.

In order to remove random noise and non-linearities while retaining the spectral components of the linear dynamics, we must guarantee that the ***magnitudes & phases*** of the linear portion of all spectral estimates are the same. This depends on how the data is sampled in each sampling window.

Single Channel Versus Multi-Channel Measurements

FFT Analyzers can be classified into two categories, ***single channel*** and ***multi-channel***. Each channel can process a unique signal. Single channel analyzers are the most popular because they cost less, but they also have limited measurement capability. The distinguishing feature of a multi-channel analyzer is that all channels are ***simultaneously sampled***. (It is also assumed that filtering and other signal conditioning match within acceptable tolerances among all channels).

If an analyzer has multiple channels, but they are ***multiplexed*** instead of simultaneously sampled, then each channel must be treated like a single channel analyzer channel.

Simultaneously sampled signals contain the correct ***magnitudes & phases relative to one another***, since they are all sampled at the same moments in time. Therefore, any two simultaneously sampled signals can be used to form a Cross Power Spectrum (**XPS**), a fundamental cross channel measurement function.

Using spectrum averaging, a single channel analyzer can remove noise and non-linearities from a spectrum if the measurement process is a ***repeatable process***. A multi-channel analyzer requires a less restrictive ***steady state*** (***stationary***) process.

Repeatable Process

In a repeatable measurement process, data acquisition must occur so that ***exactly the same time waveform*** *is* obtained in the sampling window, every time one is acquired. Figure 2 depicts a repeatable process. For a repeatable process, the ***magnitude & phase*** of each sampled signal are assumed to be unique and repeatable.

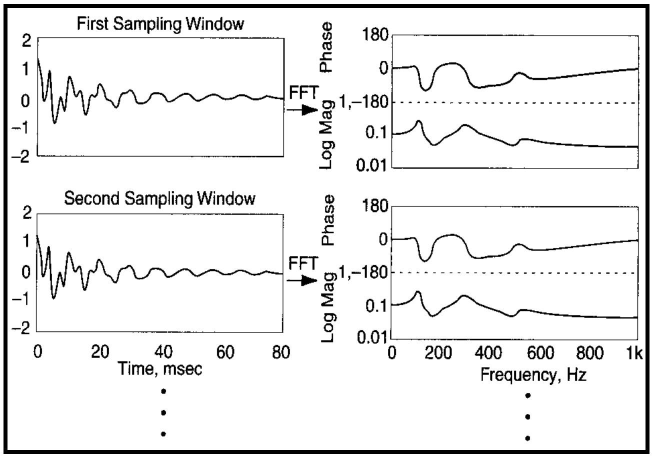

Figure 2. Repeatable Measurement Process.

A repeatable process guarantees the same results as simultaneous sampling. That is, it guarantees that multiple signals will have the ***correct magnitudes & phases relative to one another***, whether they are acquired one at a time or simultaneously. Therefore, if a repeatable measurement process can be achieved, multiple channels of data can be acquired one at a time if necessary.

To insure a repeatable process, an ***external trigger*** is usually required to capture the repeatable event in the sampling window. In machinery applications, the trigger is usually obtained as a tachometer signal from a rotating shaft.

With a repeatable process, time domain averaging can also be done to remove random noise and random non-linearities. This is also called ***synchronous averaging***.

Unfortunately, a repeatable measurement process cannot be achieved in many test situations.

Steady State Process

A steady state measurement process can be achieved in situations where a repeatable process is not achievable. A steady state process is achieved when the ***Auto Power Spectrum (APS) of a signal does not change*** from measurement to measurement. (An APS is merely the magnitude squared of an FFT, or linear spectrum.) Figure 3 shows a steady state process. Notice that the time waveform can be different in each sampling window, but its APS does not change. No special external triggering source is required for steady state measurement.

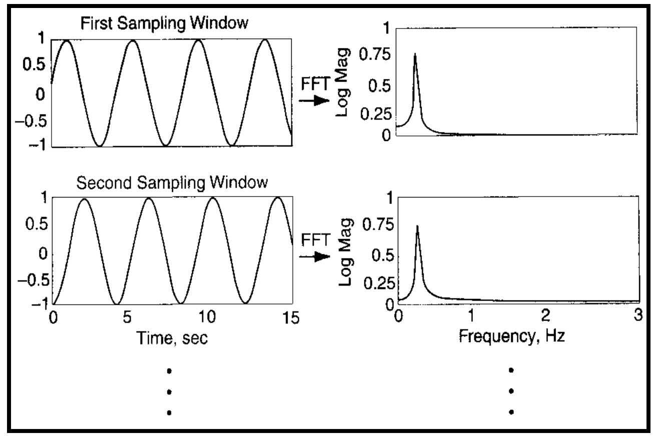

Figure 3. Steady State Measurement Process.

Tri-Spectrum Averaging

The measurement capability of a multi-channel FFT analyzer is built around a tri-spectrum averaging loop, as shown in Figure 4. This loop assumes that two or more time domain signals are simultaneously sampled. Three spectral estimates, an Auto Power Spectrum (**APS**) for each channel, and the Cross Power Spectrum (**XPS**) between the two channels, are calculated in the tri-spectrum averaging loop. After the loop has completed, a variety of other cross channel measurements (including the **FRF**), are calculated from these three basic spectral estimates.

In a multi-channel analyzer, tri-spectrum averaging can be applied to as many signal pairs as desired. Tri-spectrum averaging will remove random noise and randomly excited

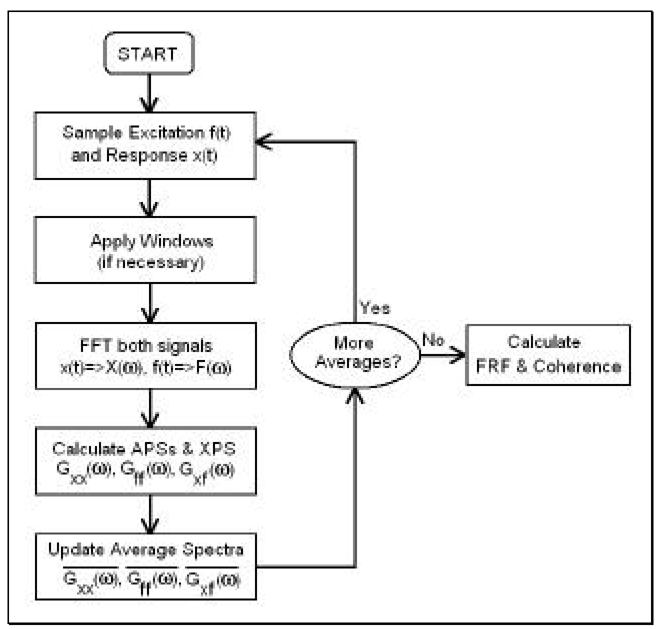

Figure 4. Tri-Spectrum Averaging Loop

non-linearities from signals taken during a steady state measurement process. This is particularly useful for measuring FRFs.

The FRF

The Frequency Response Function (FRF) is a fundamental measurement that isolates the inherent dynamic properties of mechanical structures. Experimental modal parameters (resonant frequency, damping, and mode shape) are obtained from a set of FRF measurements.

The FRF describes the input-output relationship between two points on a structure as a function of frequency, as shown in Figure 5. That is, the FRF is a measure of how much displacement, velocity, or acceleration response a structure has at an *output* point, per unit of excitation force at an *input* point.

The FRF is defined as the ratio of the Fourier transform of a motion output (or response) divided by the Fourier transform of the force input that caused the output. This is represented by the diagram in Figure 5.

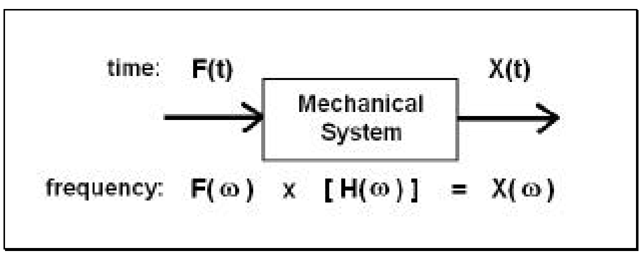

Figure 5. Block Diagram of an FRF.

Since both force and motion are vector quantities (they have directions associated with them), each FRF is actually defined between an input DOF (point and direction), and an output DOF.

An FRF is a complex valued function of frequency, that can be displayed in various forms, as shown in Figure 6.

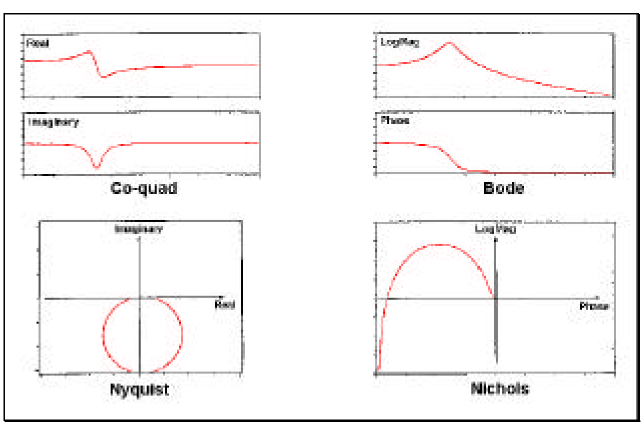

Figure 6. Alternate Forms of the FRF.

Depending on whether motion is measured as displacement, velocity, or acceleration, the FRF and its inverse have a variety of names,

- **Compliance** ⇔ displacement / force
- **Mobility** ⇔ velocity / force
- **Inertance** ⇔ acceleration / force
- **Dynamic Stiffness** ⇔ 1 / Compliance
- **Impedance** ⇔ 1 / Mobility
- **Dynamic Mass** ⇔ 1 / Inertance

On a real structure, an ***unlimited number*** of FRFs can be measured between pairs of input and output DOFs, as shown in Figure 7.

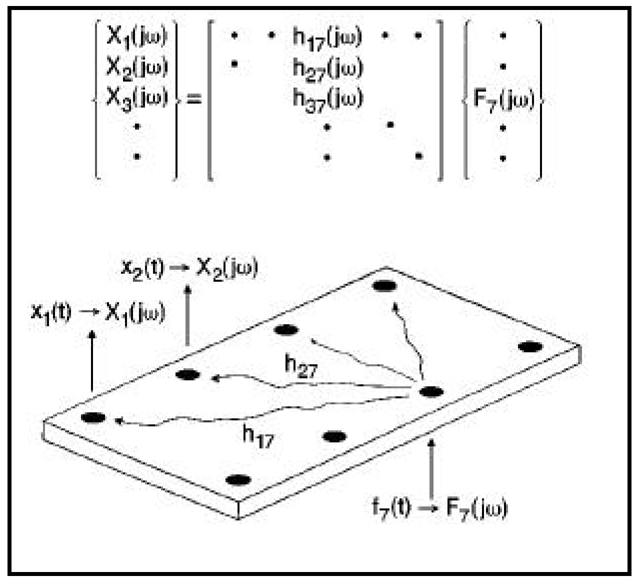

Figure 7. Measuring FRFs on a Structure

Although the FRF is defined as a ratio of Fourier transforms, is it actually computed differently using APS and XPS estimates. This is done to remove random noise and non-linearities (distortion) from the FRF, by using spectrum averaging as described earlier. There are several different ways to calculate the FRF. These are called FRF estimators.

Noise on the Output (H_1)

This FRF estimator assumes that random noise and distortion are summing into the output, but not the input of the system. For this model, the FRF is calculated as,

$$H_1 = \frac{\text{XPS}}{\text{Input APS}}$$

It can be shown that H_1 is a ***least squared error estimate*** for the FRF when extraneous noise and randomly excited non-linearities are modeled as Gaussian noise ***added to the output***.

Noise on the Input (H_2)
This FRF estimator assumes that random noise and distortion are summing into the input, but not the output of the system. For this model, the FRF is calculated as,

$$H_2 = \frac{\text{Output APS}}{\text{XPS}}$$

It can be shown that H_2 is a least squared error estimate for the FRF when extraneous noise and randomly excited non-linearities are modeled as Gaussian noise ***added to the input***.

Noise on the Input & Output (H_V)
This FRF estimator assumes that random noise and distortion are summing into both the input but and output of the system. The calculation of H_V requires more steps, and is detailed in [2].

Measuring Rows & Columns of the FRF Matrix

Structural dynamics measurement involves measuring elements from a FRF matrix model for the structure, as shown in Figure 7. This model represents the dynamics of the structure between all pairs of input and output DOFs.

FRF Matrix Model
The FRF matrix model is a frequency domain representation of a structure's linear dynamics, where linear spectra (FFTs) of multiple inputs are multiplied by elements of the FRF matrix to yield linear spectra of multiple outputs.

The FRF matrix model is written as,

$$\{\mathbf{X(\omega)}\} = [\mathbf{H(\omega)}]\{\mathbf{F(\omega)}\}$$

where:

$$\{\mathbf{X(\omega)}\} = \textbf{Linear spectra of output motions} \ldots \textbf{(n vector)}$$
$$[\mathbf{H(\omega)}] = \textbf{FRF matrix} \ldots \textbf{(n by m)}$$
$$\{\mathbf{F(\omega)}\} = \textbf{Linear spectra of input forces} \ldots \textbf{(m vector)}$$
$$\mathbf{m} = \textbf{number of inputs}$$
$$\mathbf{n} = \textbf{number of outputs}$$
$$\boldsymbol{\omega} = \textbf{frequency variable}$$

Columns of the FRF matrix correspond to inputs, and ***rows*** correspond to outputs. Each input and output corresponds to a measurement Point or DOF of the test structure.

Modal Testing
In modal testing, FRF measurements are usually made under controlled conditions, where the test structure is artificially excited by one or more shakers driven by broad band signals, or is excited by an impactor. A multi-channel FFT analyzer is then used to make FRF measurements between input and output DOF pairs on the test structure.

FRF Matrix Rows or Columns
Modal testing requires that FRFs be measured from ***at least one row or column*** of the FRF matrix. Modal frequency & damping can be obtained from any FRF measurement. A row or column of FRF measurements is required to obtain mode shapes.

When the input is fixed and FRFs are measured for multiple outputs, this corresponds to measuring elements from a ***single column*** of the FRF matrix. This is typical of a shaker test.

On the other hand, when the output is fixed and FRFs are measured for multiple inputs, this corresponds to measuring elements from a ***single row*** of the FRF matrix. This is typical of a roving hammer impact test.

Single Reference (or SIMO) Testing
The most common type of modal testing is done with either a ***single fixed input*** or a ***single fixed output***. A roving hammer impact test using a single fixed motion transducer is a common example of single reference testing. The single fixed output is called the reference in this case.

When a single fixed input (such as a shaker) is used, this is called SIMO (Single Input Multiple Output) testing. In this case, the single fixed input is called the reference.

Multiple Reference (or MIMO) Testing
When two or more fixed inputs are used, and FRFs are calculated between each of the inputs and multiple outputs, then FRFs from ***multiple columns*** of the FRF matrix are obtained. This is called Multiple Reference or MIMO (Multiple Input Multiple Output) testing. In this case, the inputs are the references.

Likewise, when two or more fixed outputs are used, and FRFs are calculated between each output and multiple inputs, this is also multiple reference testing, and the outputs are the references.

Impact Measurements

Impact testing is the most commonly used method for finding the resonances of structures and machines. A typical impact test is depicted in Figure 8.

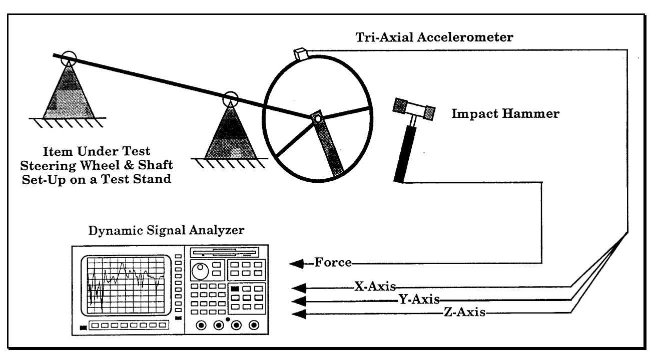

Figure 8. Impact Test Setup.

Impact testing requires a minimum of equipment,

1. A hammer with a load cell attached to its head to measure the impact force,
2. An accelerometer fixed to the structure to measure response motion,
3. A 2-channel FFT analyzer.

A wide variety of structures and machines can be impact tested. Of course, different sized hammers are necessary to provide the appropriate impact force to the structure. Not all structures can be impact tested, however. A structure or machine with delicate surfaces probably should not be impact tested. Typical signals from an impact test are shown in Figure 9.

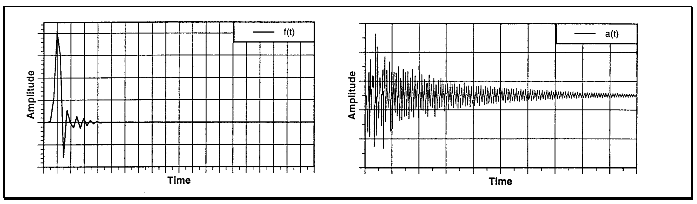

Figure 9A. Impact Force and Response Signals

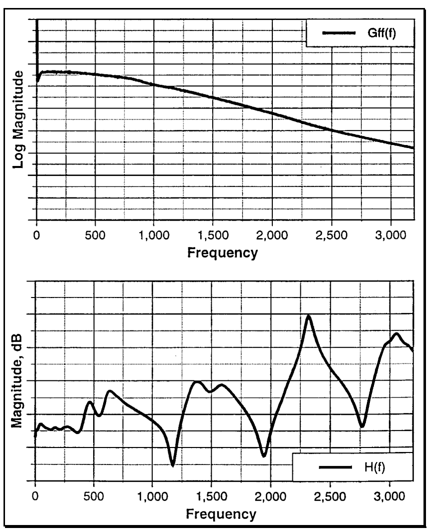

Figure 9B. Impact APS and FRF.

Roving Hammer Test

A roving hammer test is the most common type of impact test. In this test, the accelerometer is fixed at a single DOF (point and direction), and the structure is impacted at as many DOFs as desired to define the mode shapes of the structure.

Tri-axial Measurements

The only drawback to the roving hammer approach is that many points on a structure cannot be impacted in three directions, so tri-axial (3D) motion cannot be obtained for all points. When 3D motion is desired at each test point, a roving tri-axial accelerometer can be used, and the structure impacted at a fixed DOF. However, in order to process the tri-axial and force data together, however, a 4-channel FFT analyzer is required instead of a 2-channel analyzer.

Pre-Trigger Delay

Because the impulse signal exists for such a short period of time, ***it is important to capture all of it*** in the sampling window. To insure that the entire signal is captured, the analyzer must be able to capture the impulse and impulse response signals ***prior to the occurrence of the impulse***. This is called a pre-trigger delay. In other words, the analyzer must begin sampling data before the trigger point occurs, which is usually set to a small percentage of the peak value of the impulse.

Force & Exponential Windows

Two common time domain windows that are used in impact testing are the force and exponential windows. These windows are applied to the signals after they are sampled, but before the FFT is applied to them.

The force window is used to remove noise from the impulse (force) signal. Ideally, an impulse signal is non-zero for a small portion of the sampling window, and zero for the remainder of the window time period. Any non-zero data following the impulse signal in the sampling window is assumed to be measurement noise. The force window ***preserves*** the samples in the vicinity of the impulse, and ***zeros*** all of the other samples in the sampling window.

The exponential window is applied to the impulse response signal. ***The exponential window is used to reduce leakage in the spectrum of the response.*** If the response ***decays to zero*** (or near zero) before the end of the sampling window, then there will be no leakage, and the exponential window need not be used.

In the response ***does not decay*** to zero before the end of the window, then the exponential window ***must be used*** to reduce the leakage effects on the response spectrum. The exponential window ***adds artificial damping*** to all of the modes of the structure in a known manner. This artificial damping can be subtracted from the modal damping estimates. But more importantly, if the exponential window causes the impulse response to be completely contained within the sampling window, leakage is removed from its spectrum.

Accept/Reject

Because impact testing relies, to some degree, on the skill of the one doing the impacting, it should be done with spectrum averaging, using 3 to 5 impacts per measurement. Since one or two of the impacts during the measurement process may be ***bad hits***, an FFT analyzer designed for impact testing should have the ability to accept or reject each impact. An accept/reject capability saves a lot of time during impact testing since you don't have to restart the measurement after each bad hit.

Advantages of Impact Testing
The advantages of impact testing are,

- Low equipment cost.
- Ease test setup.
- Fast measurement time.
- Signals are periodic (or near periodic) in the sampling window.

Disadvantages of Impact Testing
The disadvantages of impact testing are,

- Special analyzer capabilities are required.
- Some skill required to impact correctly.
- Low energy density in impact signal.
- Doesn't remove non-linear behavior.
- Can't be used on some structures.

Shaker Measurements

When impact testing cannot be used, then structural dynamic measurements are made by providing excitation with one or more shakers attached to the structure. Common types of shakers are electro-dynamic and hydraulic shakers. A typical shaker test is depicted in Figure 10.

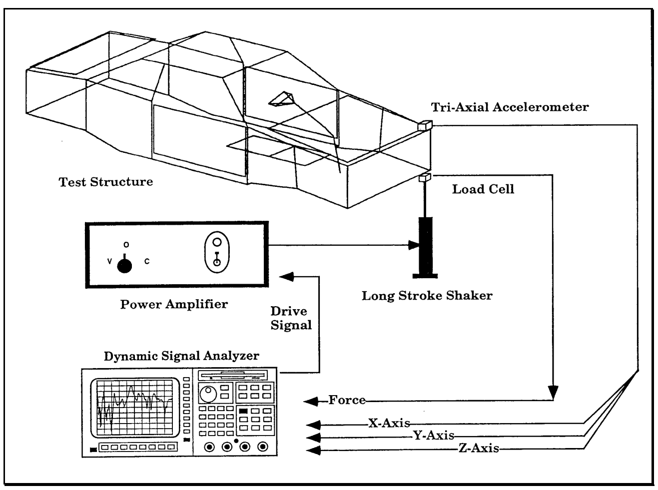

Figure 10. Shaker Test Setup.

A shaker is usually attached to the structure using a stinger (long slender rod), so that the shaker will only impart force to the structure along the axis of the stinger, the axis of force measurement.

A load cell is attached ***between the structure and the stinger*** to measure the excitation force. At least a 2-channel FFT analyzer and a uni-axial accelerometer are required to make FRF measurements using a shaker. If an analyzer with 4 or more channels is used, then a tri-axial accelerometer can be used and 3D motion of the structure measured at each test point.

In a SIMO test, one shaker is used and the shaker is the (fixed) reference. In a MIMO test, multiple shakers are used, and the shakers are the multiple references. When multiple shakers are used, care must be taken to insure that the shaker signals are not completely correlated (the same signal). Furthermore, special matrix processing software is required to calculate FRFs from the multiple input APSs and XPSs resulting from tri-spectrum averaging.

Step Sine, Slow Swept Sine
The sine wave excitation signal has been used since the early days of structural dynamic measurement. It was the only signal that could be effectively used with traditional analog instrumentation, as described earlier.

Even broad band testing methods (like impact testing), have been developed for use with FFT analyzers, sine wave excitation is still useful in some applications. The primary purpose for using a sine wave excitation signal is to put energy into a structure at a specific frequency. Slowly sweeping sine wave excitation is also useful for characterizing non-linearities in structures.

Advantages of Sine Testing
Sine wave excitation has the following advantages,

- Best signal-to-noise and RMS-to-peak ratios of any signal.
- Controlled amplitude and bandwidth.
- Useful for characterizing non-linearities.
- Long history of use.

Disadvantages of Sine Testing
The disadvantages of sine wave excitation are,

- Distortion due to over-excitation.
- Extremely slow for broad band measurements.

Broad Band Excitation Signals
A variety of new ***broad band*** excitation signals have been developed for making shaker measurements with FFT analyzers. These signals include,

- Transient
- True Random
- Pseudo Random
- Periodic Random
- Burst Random
- Fast Sine Sweep (Chirp)
- Burst Chirp

Since the FFT provides a DFT over a broad band of frequencies (***0 to nearly half of the sampling frequency***), using a broad band excitation signal makes the measurement of broad band spectral measurements much faster than using a stepped or slowly sweeping sine wave. Nevertheless, sine wave excitation is still useful in some applications.

Transient Signals
Using a transient signal in shaker testing provides the same leakage free measurements as impact testing, but with more controllability over the test. Application of the force is more

repeatable than impacting with a hand held hammer. However, this one advantage is usually outweighed by the disadvantages of using an impulsive force, when compared to the other broad band signals.

True Random
Probably the most popular excitation signal used for shaker testing with an FFT analyzer is the random signal. When used in combination with spectrum averaging, random excitation randomly excites the non-linearities in a structure, which are then removed by spectrum averaging.

A true random signal is synthesized with a random number generator, and is an unending (non-repeating) random sequence. The main disadvantage of a true random signal is that it is ***always non-periodic in the sampling window***. Therefore, a special time domain window (a Hanning window or one like it), must always be used with true random testing to minimize leakage. Typical true random signals are shown in Figure 11.

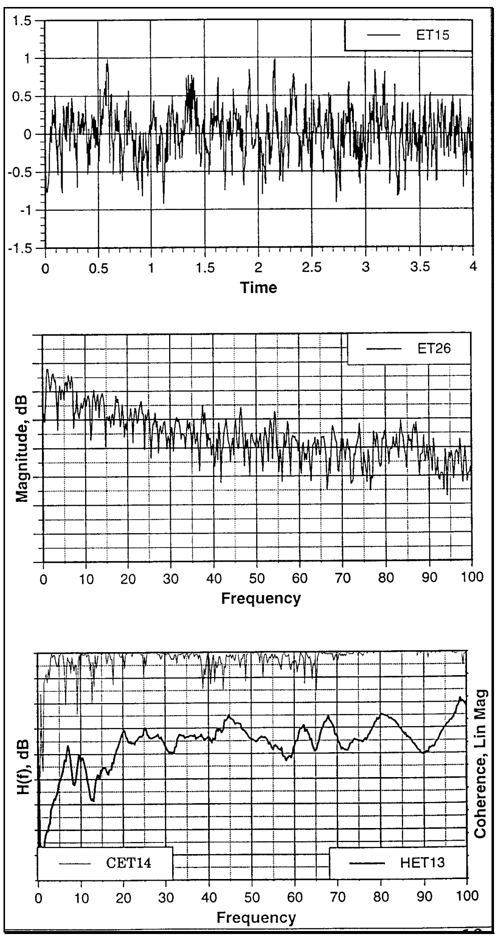

Figure 11. True Random Excitation (Time waveform, APS, FRF & Coherence).

Advantages of True Random Excitation
The advantages of true random excitation are,

- Removes non-linear behavior when used with spectrum averaging.
- Fast measurement time.
- Leakage effects reduced with Zoom measurements.

Disadvantages of True Random Excitation
The disadvantages of true random excitation are,

- Signals are non-periodic in the sampling window. Special windowing (Hanning, etc.) is needed to reduce leakage.
- Many averages are typically required.

Pseudo Random
A pseudo random signal is specially synthesized within an FFT analyzer to coincide with the DFT measurement parameters. A typical random signal starts as a uniform (or shaped) magnitude and random phase signal, synthesized over the same frequency range and samples as the intended measurement. It is then ***inverse FFT'd*** to obtain a random time domain signal, which is subsequently output through a digital-to-analog converter as the shaker excitation signal.

During the measurement process, the measured force and response signals are sampled ***over the same sampling time window*** as the output of the excitation signal. This insures that the acquired signals are ***periodic in the sampling window***, since the synthesized excitation signal is periodic in the window.

Advantages of Pseudo Random Excitation
The advantages of pseudo random excitation are,

- Signals are periodic in the sampling window, so measurements are leakage free.
- Fast measurement time.
- The amplitude of excitation can be shaped for impedance mismatches between the shaker and structure.

Disadvantages of Pseudo Random Excitation
The disadvantages of pseudo random excitation are,

- Doesn't remove non-linearities, because they are not excited randomly between spectrum averages.

Periodic Random
Periodic random excitation is simply a different use a pseudo random signal, so that non-linearities can be removed with spectrum averaging. For periodic random testing, ***a new pseudo random sequence is generated for each new spectrum average***. The advantage of this is that when multiple spectrum averages of different random signals are averaged together, randomly excited non-linearities are removed.

Although periodic random excitation overcomes the disadvantage of pseudo random excitation, it takes ***at least three times longer*** to make the same measurement. This extra time is required between spectrum averages to allow the structure to reach a new steady-state response to the new random excitation signal.

Advantages of Periodic Random Excitation
The advantages of periodic random excitation are,

- Signals are periodic in the sampling window, so measurements are leakage free.
- Removes non-linear behavior when used with spectrum averaging.
- The amplitude of excitation can be shaped for impedance mismatches between the shaker and structure.

Disadvantages of Periodic Random Excitation
The disadvantages of periodic random excitation are,

- Slower than other random test methods.
- Special software required for implementation.

Burst Random

Burst random excitation is similar to periodic random testing, but faster. In burst random testing, a true random signal can be used, but it is ***turned off*** prior to the end of the sampling window time period. This is done in order to allow the structural response to decay within the sampling window. This insures that both the excitation and response signals are completely contained within the sampling window. Hence, they are ***periodic in the sampling window***.

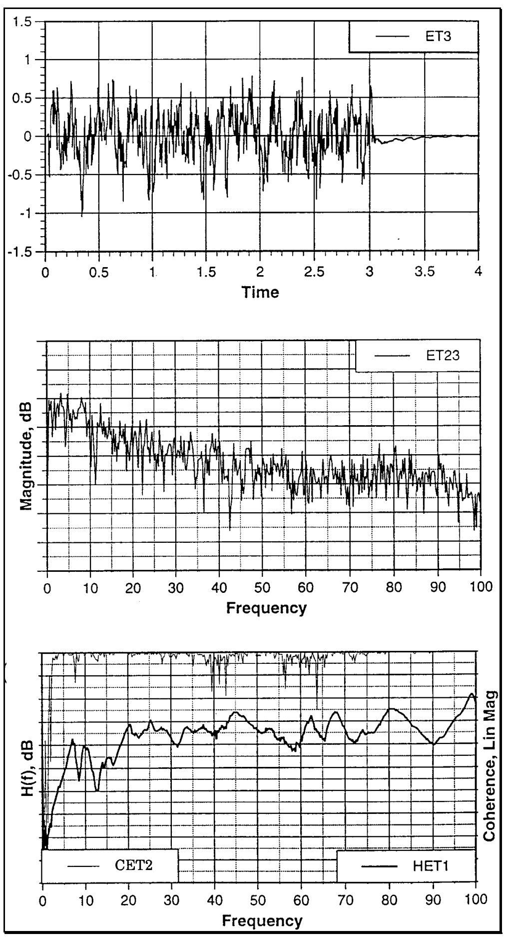

Figure 12. Burst Random Excitation (Time waveform, APS, FRF & Coherence).

Figure 12 shows a typical burst random signal. The random generator must be turned off early enough to allow the response to decay to zero (or nearly zero) before the end of the sampling window. Of course, the length of the decay period depends on the damping in the test structure.

Burst random must therefore be setup interactively on an FFT analyzer, after observing the free decay of the structure, following the removal of random excitation. Since a pure random signal can be used with burst random testing, it does not have the disadvantages of either pseudo random or periodic random testing.

Advantages of Burst Random Excitation
The advantages of burst random excitation are,

- Signals are periodic in the sampling window, so measurements are leakage free.
- Removes non-linear behavior when used with spectrum averaging.
- Fast measurement time.

Disadvantages of Burst Random Excitation
The disadvantages of true random excitation are,

- Special software required for implementation.

Chirp & Burst Chirp

A swept sine excitation signal can also be synthesized in an FFT analyzer to coincide with the parameters of the sampling window, in a manner similar to the way a pseudo random signal is synthesized. Since the sine waves must sweep from the lowest to the highest frequency in the spectrum, over the relatively short sampling window time period (**T**), this fast sine sweep often makes the test equipment sound like a bird chirping, hence the name chirp signal.

A burst chirp signal is the same as a chirp, except that it is ***turned off*** prior to the end of the sampling window, just like burst random. This is done to insure that the measured signals are ***periodic in the window***. A typical burst chirp signal is shown in Figure 13.

The advantage of burst chirp over chirp is that the structure has returned to rest before the next average of data is taken. This insures that the measured response is only caused by the measured excitation, an important requirement for any multichannel measurement such as a FRF.

Advantages of Burst Chirp Excitation
The advantages of burst chirp excitation are,

- High signal–to-noise and RMS-to-peak ratios.

- Signals are periodic in the sampling window, so measurements are leakage free.
- Fast measurement time.

Disadvantages of Burst Chirp Excitation
The disadvantages of burst chirp excitation are,

- Special software required for implementation.
- Doesn't remove non-linear behavior.

Comparison of Excitation Signals
Ideally, all of the shaker signals that are leakage free (periodic in the window) should yield the same result. Figure 14 shows an overlay of two FRF magnitudes, one measured with a burst random and the other with a burst chirp signal. The two FRFs match very well at low frequencies, but show some disparity at high frequencies. This could possibly be due to a small amount of non-linear behavior in the structure, which burst chirp signal processing cannot remove through averaging.

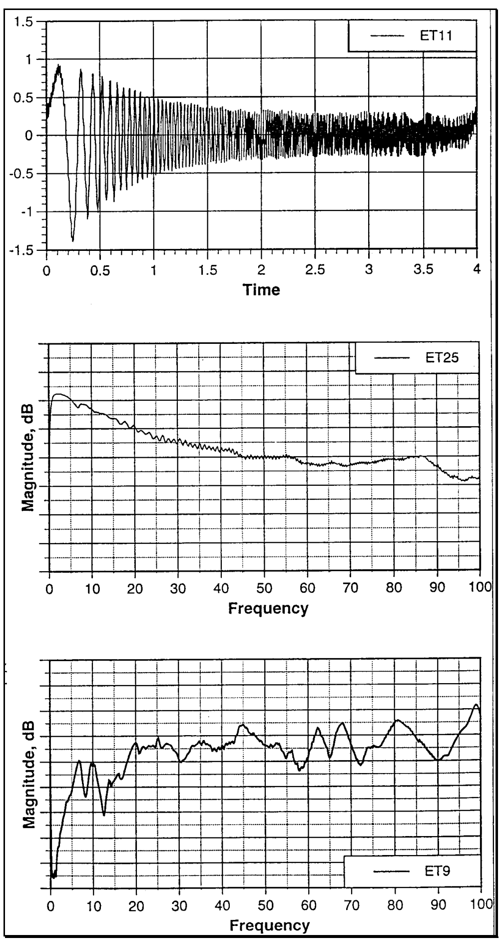

Figure 13. Burst Chirp Excitation (Time waveform, APS, FRF & Coherence)

Finally, all of the previously described test methods are compared in the table shown in Figure 15. Impact testing is by far the easiest method to implement. On the other hand, when impact testing cannot be used, or when multiple shakers are needed to provide sufficient excitation, then a variety of other implementation issues must be considered.

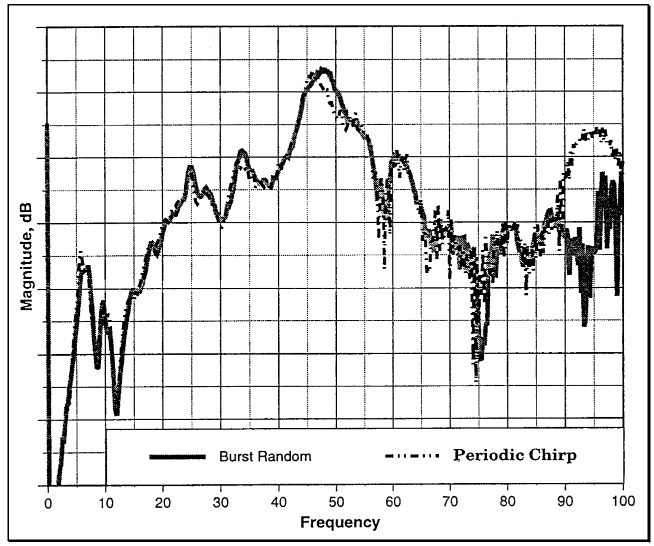

Figure 14. Burst Random Versus Burst Chirp FRF.

Difficulty with FRF Measurements

Thus far, we have talked mostly about making FRF measurements. Mode shapes (part of a resonance condition) are normally obtained from a set of FRFs. Making an FRF measurement requires that ***all of the excitation forces causing a response must be measured simultaneously with the response.*** This can be difficult, if not impossible in many test situations.

FRFs usually cannot be measured on operating machinery or equipment where ambient forces (internally generated forces, acoustic excitation, etc.) are either unmeasured or unmeasurable. On the other hand, the vibration response caused by ambient forces can always be measured, no matter what forces are causing it.

Difficulty With Operating Data Measurements
One key advantage of the FRF measurement is lost when operating data measurements are made. Without measuring the excitation forces, it is impossible to know precisely whether a peak in a response spectrum is due to a resonance or to the excitation forces. Nevertheless, valuable information can still be obtained from operating data.

	Impact	Sine	Swept Sine	True Random	Pseudo Random	Periodic Random	Burst Random	Burst Chirp
Periodic	YES	NO	YES	NO	YES	YES	YES	YES
Removes Noise	YES	NO	YES	YES	YES	YES	YES	YES
Removes Non-linearities	NO	NO	NO	YES	NO	YES	YES	NO
Test Time	FAST	SLOW	FAIR	FAIR	FAST	SLOW	FAST	FAST
SNR	LOW	HIGH	HIGH	FAIR	FAIR	FAIR	FAIR	HIGH
Frequency Control	SOME	YES	YES	YES	YES	YES	YES	YES

Figure 15. Comparison of Excitation Methods

Operating Deflection Shapes

An Operating Deflection Shape (ODS) is defined as any ***forced motion of two or more*** DOFs on a structure. Specifying the motion of two or more DOFs defines a shape. Stated differently, a shape is the motion of one DOF relative to all others.

An ODS can be defined from any forced motion, either at a moment in time, or at a specific frequency. An ODS can be obtained from different types of time domain responses, be they random, impulsive, or sinusoidal. An ODS can also be obtained from many different types of frequency domain measurements, including linear spectra (FFTs), APS, XPSs, FRFs, transmissibilities, and a special type of measurement called an ODS FRF, described later.

Mode Shapes and ODSs Contrasted

Mode shapes and ODSs are related to one another, but have different characteristics,

1. Modes are inherent properties of a structure. They don't depend on the forces or loads acting on the structure.
2. Modes will change if the material properties (mass, stiffness, damping properties), or boundary conditions (mountings) of the structure change.
3. Mode shapes don't have unique values, and hence don't have units associated with them.
4. Mode shapes are unique. That is, the motion of one DOF relative to all others at resonance is unique.
5. Modes are defined for linear, stationary systems.
6. Modes are only used to characterize resonant vibration.

ODSs have the following characteristics

1. ODSs depend on the forces or loads applied to a structure. They will change if the load changes.
2. ODSs also depend on the modes. ODSs will change if the modes change.
3. ODSs have unique values & units, typically displacement, velocity, or acceleration, or perhaps displacement per unit of excitation force.
4. ODSs can be used to answer the question, ***"How much is the structure really moving, at a particular time or frequency?"***
5. ODSs can be defined for nonlinear or non-stationary structures.
6. ODSs can also be defined for structures that don't resonate.

Modes From ODSs

Since all measurement data is forced response, whenever two of more measurements are taken spatially from two or more DOFs of a structure, this is an ODS measurement. Moreover,

- ***All experimental modal parameters are obtained by post-processing ODS measurements!***

Transmissibility Measurement

We have already seen that under the assumption of either a Repeatable (more restrictive) or a Steady State (less restrictive) measurement process, spectrum averaging can be accomplished, and multi-channel measurements made. When the excitation force cannot be measured, then a ***reference response signal*** can be used instead of the force.

A transmissibility measurement is calculated in the same way as an FRF, but with a reference response signal replacing the excitation force.

Mode Shapes From Transmissibilities

A set of transmissibilities, calculated between multiple response DOFs and a single fixed reference response, can be used to find the mode shapes of structural resonances. The ***values of the transmissibilities at each resonant frequency*** is an ***approximation to the mode shape***.

The difficulty with using a set of transmissibilities to determine mode shapes is that resonances correspond to "flat spots" instead of peaks in these measurements. Therefore, in addition to a set of transmissibilities, at least one APS is required in order to locate the resonance peaks.

ODS FRF Measurement

A different cross channel measurement, called and ODS FRF, can be calculated from APS and XPS measurements [3]. An ODS FRF has two advantages over a transmissibility,

1. ***It has peaks at resonant frequencies***, making it easier to locate resonances and identify mode shapes.
2. It has responses units (G's, Mils, etc.). Therefore, operating deflection shapes taken from a set of ODS FRFs have these same units.

To calculate a set of ODS FRFs between multiple response DOFs and a single fixed ***reference response***, a tri-spectrum averaging loop is used to estimate an APS for each response, and a XPS between each response and the reference response. When tri-spectrum averaging is completed, each ODS FRF is formed by ***replacing the magnitude of the each XPS with the APS*** of its corresponding response.

A set of ODS FRF measurements is useful for determining whether a structure or machine is simply undergoing excessive forced response, or whether a resonance is also being excited.

Non-Steady State Operation

All of the foregoing measurements assumed that measurement process was either repeatable or steady state. However, many types of structures and machines undergo non-steady state operation. Automobiles and machine tools are common examples.

In fact, most rotating equipment is characterized by non-steady state operation. Measurements are typically made while sweeping the speed of the machine. These are called RPM sweeps. Since the measurement process is non-steady state, the spectra cannot be averaged together. Rather, they are plotted in a waterfall plot, or spectral map.

Orders

Since the excitation forces in a rotating machine are primarily sinusoidal and usually cannot be measured, their response spectra will exhibit forced responses are peaks that vary in frequency with the speed of the machine. These peaks, called Orders, appear at frequencies that are fixed multiples of the machine speed.

Since machine speed continually changes (it is non-stationary), a portion of a rotating machine's response will also be non-stationary and exhibit peaks which "track" the cyclic forces. However, if a resonance is excited, it will always appear at its fixed (stationary) natural frequency in any spectral measurement.

Conclusions

During the past 30 years, there has been a proliferation of new structural dynamics testing methods that are based upon the laboratory implementation of the FFT and related signal processing algorithms. The "parallel processing" nature of the FFT which yields the discrete frequency spectrum of a signal from one calculation, makes it an ideal tool for broad band testing of structures. This created a fundamental departure from the traditional sine wave based, swept filter methods for testing structures.

For finding structural resonances, the FFT has made it convenient to excite structures using many different kinds of broad band signals. Not only are a variety of shaker signals now used, but impact testing has became very popular as a fast, convenient, and relatively low cost way of finding the mode shapes of structures.

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