GLOBAL FREQUENCY & DAMPING ESTIMATES FROM FREQUENCY RESPONSE MEASUREMENTS

by
Mark H. Richardson
Structural Measurement Systems, Inc.
San Jose, California

ABSTRACT

In two previous IMAC papers ([1] & [2]), the Rational Fraction Polynomial method for estimating modal parameters from Frequency Response Functions (FRFs) was introduced and discussed. Also, the concepts of Local and Global curve fitting were presented, and comparisons between these two approaches were given.

In this paper, a new formulation of the Rational Fraction Polynomial equations is presented which will obtain ***global estimates*** *of modal frequency and damping from a set of FRF measurements. This algorithm can then be used in conjunction with the previously described Global Residue algorithm [2] to obtain frequency, damping, and mode shape parameters from a set of FRF measurements.*

This paper includes some examples of the use of Global curve fitting, as well as a discussion of the advantages & disadvantages of Local versus Global versus Poly-Reference curve fitting. Also, the problem of compensating for the effects of out-of-band modes is covered, and the unique way in which this new Global method can handle these effects is illustrated.

INTRODUCTION

Modes of vibration are global properties of a structure. That is, a mode is defined by its natural frequency, damping, and mode shape, each of which can be measured (or estimated) from a set of FRF measurements that are taken from the structure.

Modal properties are evidenced by resonance peaks which appear in the FRF measurements. Modal frequency is closely related to the frequency of a resonance peak, and is often approximated by using the peak frequency itself. Modal damping is evidenced by the width of the resonance peak, and can be approximated as one-half of the difference between the two frequencies on either side of the peak where the FRF value is equal to .707 of the peak value. These two points are known as the Half Power Points. (See Figure 1).

Mode shapes are evidenced by the heights of the resonance peaks, and are commonly obtained by assembling the resonance peak values from a set of FRF measurements.

In summary then, modal frequency and damping can be measured from any FRF measurement, except those where the resonance peak has zero amplitude. Each mode is evidenced by a resonance peak which has the same peak frequency and width in every FRF measurement. Each mode shape is measured by assembling the peak values of each of the resonance peaks which occur at the same frequency in all of the FRF measurements, as shown in Figure 1. If there is no peak in a particular measurement but the peak appears in other measurements, the mode is said to be at a node point, i.e. its mode shape amplitude is zero.

CURVE FITTING FRFs

Modal properties can also be defined as parameters of a linear dynamic mathematical model of a structure.

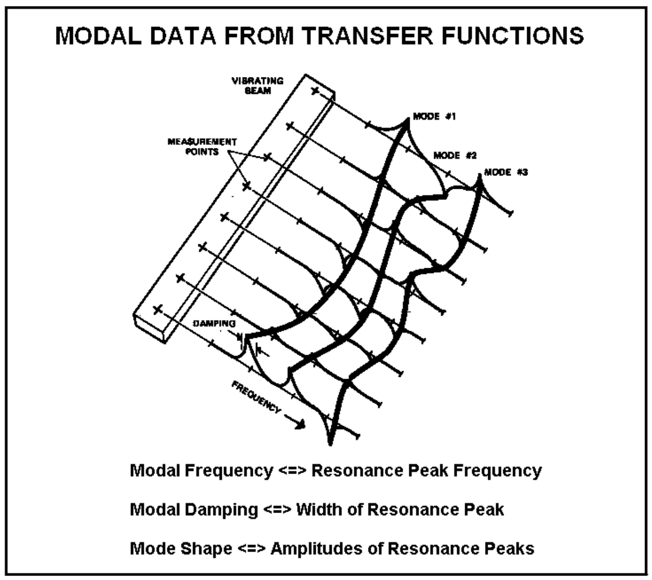


FIGURE 1

This can be done with either a time domain model (a set of differential equations), or an equivalent frequency domain model (a transfer function matrix). In this paper, a form of the FRF matrix model will be used since the FRF is simply a special case of the transfer function. (See figures 2 & 3).

Regardless of which model is used, all modal parameter estimation techniques attempt to match up an analytical expression of the structure's dynamics with some measured data which represents its dynamics. This process of matching up an analytical function with a set of measured data is called curve fitting. During the process of curve fitting, the unknown parameters of the model are estimated, hence the process is also called **parameter estimation**.

The purpose of all curve fitting in modal analysis, then, is to obtain the most accurate estimates of modal parameters as possible. Unfortunately, it is possible with many curve fitters to obtain good curve fits, i.e. match the analytical model to the measured data very closely, but still obtain poor estimates of the modal parameters. Another way of saying this is that good curve fits are necessary for accurate parameter estimates, but not sufficient.

LOCAL VERSUS GLOBAL CURVE FITTING

Most curve fitting is done today in a 'local' sense. That is, each measurement is individually curve fit, and the modal properties for each mode are estimated.

Four parameters (frequency, damping, and complex residue) are estimated for each mode, and if an MDOF fitter is used, a total of 4 x (number of modes) unknowns are estimated simultaneously from each measurement. With a large number of unknowns, significant errors can occur in the resulting estimates, even though the curve fits look good.

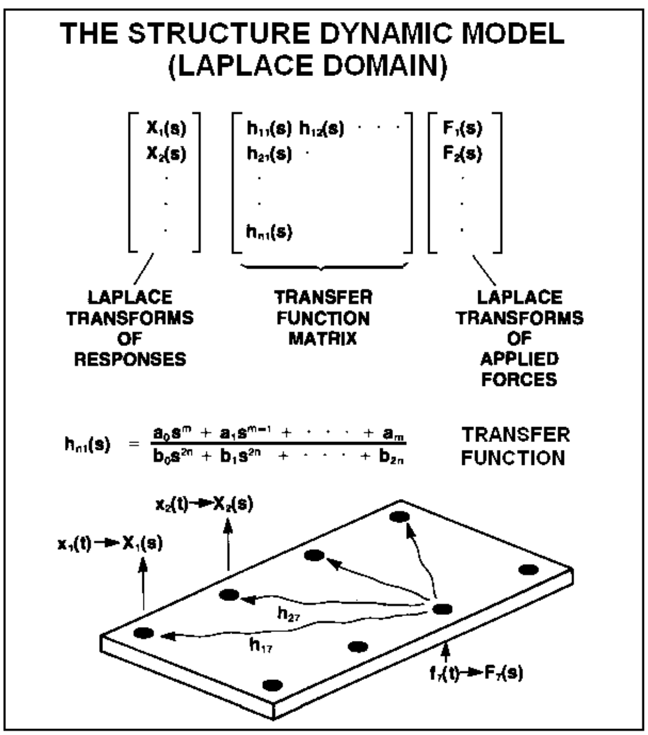


FIGURE 2

One approach that can reduce errors is to divide the curve fitting process into two steps:

(1) estimate modal frequency and damping using all measurements
(2) estimate residues using the fixed frequency and damping values.

This process is called Global Curve Fitting. In a previous paper, an algorithm was introduced for performing step (2). In the next section, a new algorithm for performing step (1), i.e. obtaining estimates of global frequency and damping, is introduced.

A GLOBAL RFP METHOD FOR ESTIMATING FREQUENCY & DAMPING

The Rational Fraction Polynomial (RFP) method was originally introduced in reference [1]. In that paper, it was shown how the characteristic polynomial could be identified from multiple measurements. The use of orthogonal polynomials uncouples the solution equations of the RFP method so that the denominator (characteristic) polynomial coefficients can be obtained independently of the numerator polynomial coefficients.

Taking advantage of the fact that the characteristic polynomial is the same for all FRF measurements which are taken from the same structure, the solution equations for the characteristic polynomial coefficients can be written for as many measurements as desired. This is shown in Figure 19 of reference [1], and is repeated here in Figure 4. Expression (1) in Figure 4 contains the repeated application of the solution equations to (**p**) different FRF measurements.

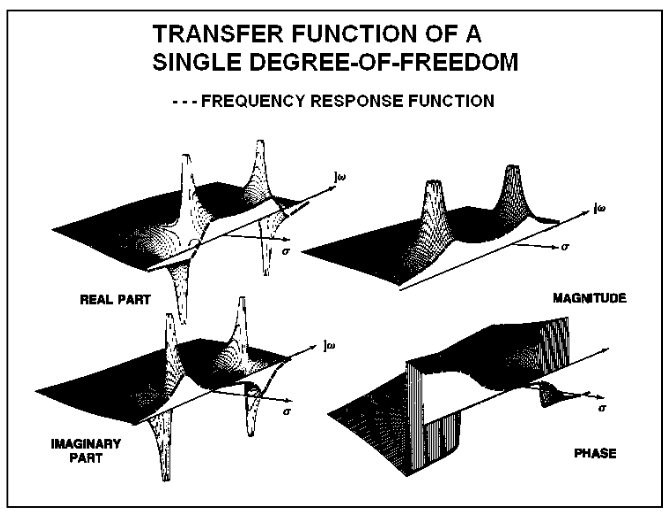


FIGURE 3

Characteristic Polynomial from Multiple Measurements

$$\begin{bmatrix} \mathbf{U}_1 \\ \cdots \\ \mathbf{U}_2 \\ \cdots \\ \vdots \\ \cdots \\ \mathbf{U_p} \end{bmatrix} \{\mathbf{B}\} = \begin{Bmatrix} \mathbf{V}_1 \\ \cdots \\ \mathbf{V}_2 \\ \cdots \\ \vdots \\ \cdots \\ \mathbf{V_p} \end{Bmatrix} \qquad \mathbf{(1)}$$

where

$$[\mathbf{U_k}] = [\mathbf{I} - [\mathbf{X_k}]^{\mathbf{t}}[\mathbf{X_k}]][\mathbf{BM_k}]^{-1} \qquad \mathbf{(n\ x\ n)}$$

$$\{\mathbf{V_k}\} = [\mathbf{X_k}]^{\mathbf{t}}\{\mathbf{H_k}\} \qquad \mathbf{(n\text{-}vector)}$$

$$[\mathbf{X_k}] = -\mathbf{Re}\left(2[\mathbf{P_k}^*]^{\mathbf{t}}[\mathbf{T_k}]\right) \qquad \mathbf{(m+1\ x\ n)}$$

$$\{\mathbf{H_k}\} = \mathbf{Re}\left(2[\mathbf{P_k}^*]^{\mathbf{t}}[\mathbf{W_k}]\right) \qquad \mathbf{(m+1\text{-}vector)}$$

$[\mathbf{P_k}] =$ numer. polynomials **(L x m+1)**

$[\mathbf{T_k}] =$ denom. polynomials **(L x n)**

$[\mathbf{W_k}] =$ denom. polys & data **(L-vector)**

$[\mathbf{BM_k}] =$ orthogonal to ordinary transform

Least Squared Error Solution

$$\sum_{\mathbf{k}=1}^{\mathbf{p}} [\mathbf{U_k}]^2\{\mathbf{B}\} = \sum_{\mathbf{k}=1}^{\mathbf{p}} [\mathbf{U_k}]\{\mathbf{V_k}\} \qquad \mathbf{(2)}$$

$\{\mathbf{B}\} =$ char. polynomial coeffs. (n-vector)

m= order of the numerator polynomial

n = order of the denominator polynomial

L = no. of FRF data points used.

FIGURE 4

If there are (**n**) unknown polynomial coefficients in the n-vector (**B**), then there are (**n by p**) equations in expression (1). This is an over specified set of equations since only (**n**) equations are needed to solve for the unknowns. Hence a "least squared error" set of equations (2) is solved instead. These equations can use data from any desired number of FRF measurements, but will always solve for the same number of unknowns (**B**).

Reference [1] contains an error in that the solution equations are written in Figure l9 with the orthogonal polynomial coefficients (**D**) as unknowns. These coefficients are not the same for all measurements, but the ordinary polynomial coefficients (**B**) are the same. So, the equations must be written in terms of the ordinary polynomial coefficients, as shown in figure 4. Note that the orthogonal coefficients are related to the ordinary coefficients by a known invertable matrix [$\mathbf{BM_k}$].

Note also that the [$\mathbf{X_k}$] matrix is made up of matrices that contain both numerator [$\mathbf{P_k}$] and denominator [$\mathbf{T_k}$] orthogonal polynomials. (See reference [1] for details). This allows us to add in extra numerator terms to the curve fitting model as a means of compensating for the effects of out-of band modes. The advantage of this capability, which is unique to the RFP method, with be illustrated later on by example.

The Global RFP Frequency & Damping algorithm consists essentially of setting up and solving equations (2). To set up the process, the operator must specify the total number of modes, the frequency band of FRF measurement data to be used, and the number of extra numerator polynomial terms, if desired. After equations (2) are solved and the polynomial coefficients determined, they are passed into a polynomial root solver which finds the global frequency and damping estimates.

TEST CASE WITH HEAVY MODAL COUPLING

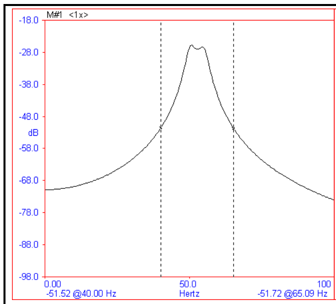
In this first example, the Global RFP method is compared to the Local RFP method by curve fitting five synthesized, noise contaminated, FRF measurements which contain three heavily coupled modes. Plots of the log magnitudes of these measurements are shown in Figure 5, along with a listing of the modal parameters used to synthesize them. (Only the measurement data between 45 Hz and 60 Hz, as indicated with line cursors on the measurement plots, was used for curve fitting).

These measurements contain some conditions which are oftentimes difficult for Local fitters to handle. Notice that measurements #1 and #3 contain node points for modes #2 and #1 respectively. Notice also that measurements #4 and #5 are characteristic of driving point measurements (all residues have the same sign) which are typically difficult to curve fit.

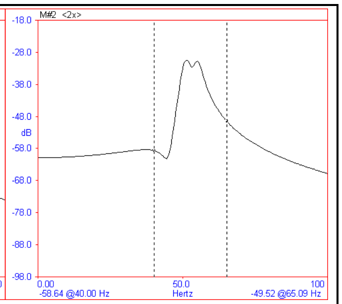
Modal Data Used To Synthesize Measurements

Mode No.:	**1**	**2**	**3**
Frequency (Hz):	**50.0**	**52.0**	**55.0**
Damping (%):	**3.0**	**2.5**	**3.0**
		------- Residues ---------	
Meas. No.1:	**1.0**	**0.0**	**-1.0**
Meas. No.2:	**0.5**	**-0.25**	**-0.6**
Meas. No.3:	**0.0**	**-0.5**	**0.6**
Meas. No.4:	**-0.5**	**-0.75**	**-0.3**
Meas. No.5:	**-1.0**	**-1.0**	**-0.8**

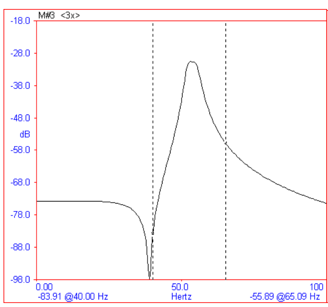
FIGURE 5.a



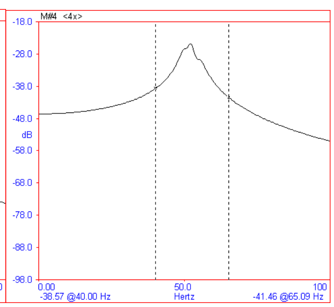
Meas. No.1



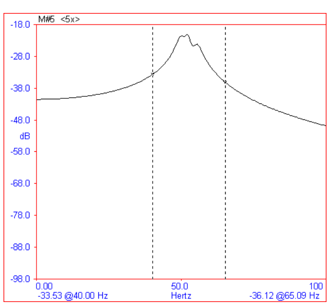
Meas. No.2



Meas. No.3



Meas. No.4



Meas. No.5

FIGURE 5.b

Figure 6 shows the results of curve fitting these measurements with a Local RFP fitter. At least two problems can be pointed out in these results:

Local Rational Fraction Polynomial Results

Meas. No.1 (Identified first two modes correctly)

Mode	Freq(Hz)	Damp(%)	Mag	Phs
1	49.99	3.03	1.998E+00	.33
2	55.01	3.00	1.918E+00	180.20
3	55.26	.95	7.495E-03	43.96

Meas. No. 2 (Large damping & Residues errors)

MODE	FREQ(Hz)	DAMP(%)	AMPL	PHS
1	50.09	2.38	4.197E-01	339.91
2	54.72	4.26	7.120E-01	146.28
3	55.60	.73	4.585E-02	185.47

Meas. No.3 (Modes assigned to wrong mode nos.)

MODE	FREQ(Hz)	DAMP(%)	AMPL	PHS
1	51.99	2.49	4.914E-01	100.73
2	55.00	2.92	5.873E-01	.73
3	57.77	.95	7.943E-04	7.43

Meas. No.4 (Large damping & Residues errors)

MODE	FREQ(Hz)	DAMP(%)	AMPL	PHS
1	49.92	1.36	1.174E-01	207.24
2	51.83	4.04	1.468E+00	178.87
3	56.14	1.07	8.045E-02	167.07

Meas. No.5 (Large damping & Residues errors)

MODE	FREQ(Hz)	DAMP(%)	AMPL	PHS
1	50.12	2.61	9.644E-01	109.27
2	52.16	2.99	1.448E+00	190.45
3	55.66	2.28	4.929E-01	175.28

FIGURE 6

Problem #1: Notice that the Local fitter obtained the correct results for measurements #1 and #3. In each case a third mode was also found but the amplitude of its residue was very small, making it insignificant. The problem occurred, though, in assigning the parameters to the mode numbers. When node points are encountered like this, there is no straightforward way to sort out the results of a Local fitter and assign the parameters to the correct mode.

Problem #2: The Local fitter was unable to obtain accurate parameter estimates for measurements #2, #4, and #5, where all three modes were present. The best clue of this is in the wide disparity of damping estimates in each measurement, even though the correct answers are close in value. Damping is the most difficult modal parameter to estimate, and in a closely coupled case like this, there will often be large variations in its estimates from one measurement to the next.

The Global RFP Frequency & Damping results are shown in figure 7. In Case #1, only the first measurement was used. This measurement only has two modes present, but the curve fitter was told there were three. The negative damping estimate for mode #2 indicates that only two valid modes were found.

Global Frequency and Damping Estimates

Case No.1 (Using Meas. No.1 only)

Mode	Freq(Hz)	Damp(%)	Damp(Hz)
1	50.00	2.98	1.49
2	52.70	-.00	-.03
3	55.03	2.96	1.63

Case No. 2 (Using Meas. No.1 & No.2)

Mode	Freq(Hz)	Damp(%)	Damp(Hz)
1	49.99	2.95	1.48
2	51.97	2.71	1.41
3	55.03	3.01	1.66

Case No.3 (Using Meas. No.1, No.2, & No.3)

Mode	Freq(Hz)	Damp(%)	Damp(Hz)
1	50.00	2.97	1.48
2	52.01	2.67	1.39
3	55.02	3.02	1.66

Case No.4 (Using all 5 Measurements)

Mode	Freq(Hz)	Damp(%)	Damp(Hz)
1	49.96	2.93	1.47
2	52.05	2.60	1.35
3	55.04	2.98	1.64

FIGURE 7

In Case #2, measurements #1 and #2 were used. Since measurement #2 contains all three modes, valid estimates of frequency and damping for all three modes were obtained.

In Case #3, the first three measurements were used, and in Case #4, all five measurements were used. All of the estimates in Case #4 are in error by less than 5%, which is extremely good considering the amount of noise, modal coupling, and the two node points of these measurements.

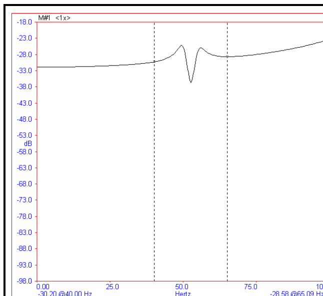
COMPENSATION FOR OUT-OF-BAND MODES

FRF measurements usually contain the residual effects of modes which lie outside of the measurement frequency range. In addition, curve fitting is usually always done in a more limited frequency range which surrounds the resonance peaks of the modes of interest.

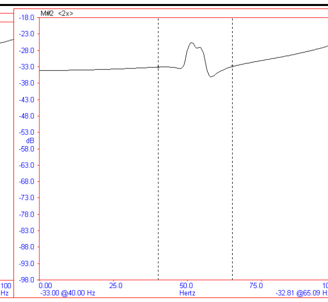
Synthesize Measurements with an Out-of-Band Mode

Mode No.:	1	2	3	4
Frequency (Hz):	50.0	52.0	55.0	125.0
Damping (%):	3.0	2.5	3.0	4.0
	-------Residues	---------		
Meas. No.1:	1.0	0.0	-1.0	20.0
Meas. No.2:	0.5	-0.25	-0.6	-15.0
Meas. No.3:	0.0	-0.5	0.6	20.0
Meas. No.4:	-0.5	-0.75	-0.3	-18.0
Meas. No.5:	-1.0	-1.0	-0.8	-10.0

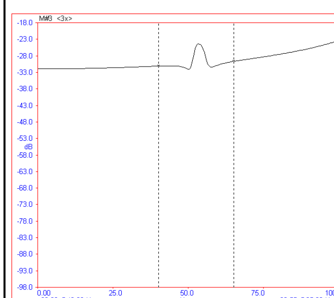
FIGURE 8.a



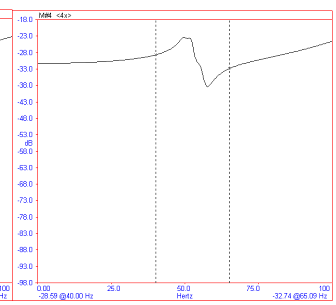
Meas. No.1



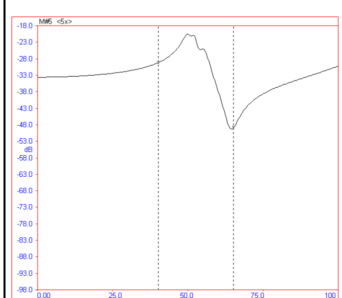
Meas. No.2



Meas. No.3



Meas. No.4



Meas. No.5

FIGURE 8.b

Hence, to obtain accurate parameter estimates, all curve fitters must somehow compensate for the residual effects of modes which lie outside of the curve fitting band.

With an MDOF curve fitter, you can always over specify the number of modes to be used in the curve fitting process, and this is commonly done to compensate for out-of-band modes. However, after the curve fitting is done, the parameters of so-called *computational modes* must be sorted out and removed from the desired results. Operator judgment is usually required to sort out the computational modes.

The RFP method has the advantage that additional numerator polynomial terms can be added to compensate for out-of-band modes, thus avoiding the problem of computational modes.

Illustrative Example: Figure 8 shows five synthesized, noise contaminated, measurements which were synthesized with the same modal parameters as those in Figure 5, but with a fourth out-of-band mode added at 125 Hz.

Global Frequency and Damping Estimates with Out-of-Band Mode

Case No.1 (Using no extra polynomial terms)

Mode	Freq(Hz)	Damp(%)	Damp(Hz)
1	47.64	20.95	10.21
2	51.76	2.73	1.42
3	55.12	2.57	1.42

Case No. 2 (Using 4 modes, Mode No.4 is computational)

Mode	Freq(Hz)	Damp(%)	Damp(Hz)
1	49.99	2.94	1.47
2	52.01	2.61	1.36
3	55.05	3.91	1.00
4	117.32	-4.58	-5.38

Case No.3 (Using 4 extra polynomial terms)

Mode	Freq(Hz)	Damp(%)	Damp(Hz)
1	49.98	3.00	1.50
2	52.09	2.48	1.29
3	55.06	2.63	1.45

FIGURE 9

Figure 9 shows the results which were obtained from using the Global RFP fitter in Case #1: with no compensation, Case #2: with an extra mode, and Case #3: with 4 extra numerator terms. (Again, only the data between 45 Hz and 60 Hz was used by the curve filter).

Adding an extra mode worked well in this case because, of course, there was a fourth mode in the data. However, in most cases there will be complex combinations of many out-of-band modes causing residual errors within the band.

Case #3 shows that the use of extra numerator terms adequately compensates for the residual effects of out-of-band modes, and eliminates the problem of sorting out and removing the parameters of "computational" modes from the curve fitting results.

COMPARISON OF CURVE FITTING METHODS

In this paper and a previous one [2], we have shown some of the advantages of Global curve fitting over Local curve fitting, specifically for the MDOF case. One might rightfully ask, though, “What about SDOF methods?”, or "What about the newer multiple reference, (or Poly-Reference) methods?” “Where and when are these methods used?".

In Figure 10, I attempt to answer these questions by showing a comparison of all of the popular curve fitting methods as to their advantages and disadvantages.

For troubleshooting work, and with lightly coupled modes, SDOF methods are still preferred to all others because they are fast and don't require a lot of operator skill.

Local MDOF methods will generally obtain better results than SDOF methods with heavy noise and modal coupling, but the operator must be careful to "steer" the fitter through local modes and node point situations.

Global fitting offers definite advantages over Local MDOF fitters, as already pointed out here, but there are still some cases, e.g. repeated roots, that Global fitters cannot handle.

The newly developed Poly-Reference algorithms ([3], [5]) offer even greater promise for obtaining accurate modal parameter estimates by curve fitting FRF measurements. These algorithms can simultaneously handle data from two or more rows or columns of the FRF matrix, i.e. multiple references. Hence, situations such as repeated roots, which require two or more rows or columns of data to resolve, can be handled with a Poly-Ref fitter. In addition, because several references are used, the chances of missing a mode of the structure are lessened.

Of course, in a large modal test, it is always advisable to test the structure using several reference points to insure that no modes are overlooked. Local or Global fitters can be used on multiple reference FRF data as well.

On a related subject [4], the real advantage of multiple shaker (simultaneous multiple reference) testing, is that large structures can be more effectively excited and non-linearity’s removed so that better quality FRF measurements are made. Any type of curve fitter; Local, Global, or Poly-Reference, can be used on these measurements also. In seeking accurate modal parameter estimates, the primary consideration, regardless of which type of curve fitter is used, should always be to obtain the highest quality FRF measurements possible.

Comparison of Curve Fitting Methods

Method	Advantages	Disadvantages
Local SDOF	∗**Fast** ∗**Easy to use**	∗**Only for light modal coupling.**
Local MDOF	∗**Good with heavy modal coupling** ∗**Good with noise**	∗**Skill required to choose No. of modes.** ∗**Poor results will local modes & nodal points.**
Global	∗**Global Freq & Damp Est.** ∗**Good with local modes & nodal points.**	∗**Errors with Freq or Damp variations.** ∗**Errors with poorly excited modes.**
PolyReference	∗**Good for repeated roots.**	∗**Poor results with inconsistent measmts.**

FIGURE 10

CONCLUSIONS

It was shown in this paper that Global curve fitting offers some distinct advantages over Local curve fitting. However, for simple structures with lightly coupled modes, Local methods will still yield acceptable results. On the other hand, for larger more complex structures with heavy modal coupling, Global or Poly-Ref curve fitters should always give superior results to those of Local fitters.

A remaining problem with the use of all current-day curve fitters is that the operator must still determine before-hand how many modes are in the data. Research is being done, however, on new algorithms which will automatically determine how many modes are present, and some recent results are very encouraging.

REFERENCES

[1] Richardson, M. H. & Formenti, D. L. "Parameter Estimation from Frequency Response Measurements using Rational Fraction Polynomials", Proceedings of the 1st International Modal Analysis Conference, Orlando. Florida, November 8-10, 1982.

[2] Richardson, M. H. & Formenti, D. L. "Global Curve Fitting of Frequency Response Measurements using the Rational Fraction Polynomial Method", Proceedings of the 3rd International Modal Analysis Conference, Orlando, Florida, January 28-31, 1985.

[3] Hunt, D. L., Rocklin, **G.** T., Crowley, **J.** R. & Vold, H. **"**The Practical Use of the Polyreference Modal Parameter Estimation Method", Proceedings of the 2nd International Modal Analysis Conference, Orlando, Florida, February 6-9, 1984.

[4] Van Karsen, C. & Allemang, R. J. "Single and Multiple Input Modal Analysis. **A** Comparison of Averaging Techniques and Results", Proceedings of the 2nd International Modal Analysis Conference, Orlando. Florida, February 8-9, 1984.

[5] Zhang, L. & Kanda, H. "The Algorithm and Application of a New Multi-input Multi-output Modal Parameter Identification Method", 56th Shock and Vibration Symposium. U.S. Naval Postgraduate School, Monterey, California, October 22-24, 1985.